Quantum error correction and feedback

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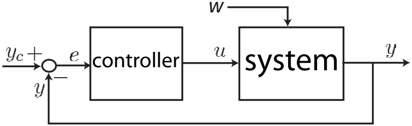
Underlying issues

Quantum Error Correction (QEC) is based on a discrete-time feedback loop

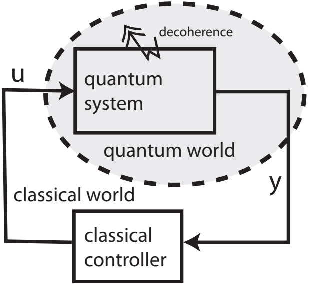
- A typical stabilizing feedback-loop for a classical system



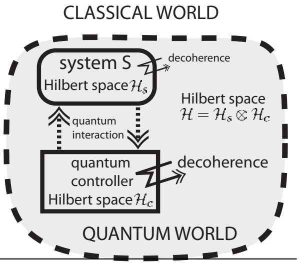
- Current experiments: 10^{-3} (soon 10^{-4}) is the typical error probability during elementary gates (manipulations) involving few physical qubits.
- High-order error-correcting codes with an important overhead;
- Today, no such controllable logical qubit has been built.
- Key issue: reduction by several magnitude orders of such error rates, far below the threshold required by actual QEC, to build a controllable logical qubit encoded in a reasonable number of physical qubits and protected by QEC.

Control engineering can play a crucial role to build a controllable logical qubit protected by **adapted feedback schemes increasing precision and stability.**

Two kinds of quantum feedback[1]



Measurement-based feedback: controller is classical; measurement back-action on the quantum system of Hilbert space $\mathcal{H}$ is stochastic (collapse of the wave-packet); the measured output y is a classical signal; the control input u is a classical variable appearing in some controlled Schrödinger equation; $u(t)$ depends on the past measurements $y(\tau)$, $\tau \leq t$.



Coherent/autonomous feedback and reservoir/dissipation engineering: the **system of Hilbert space $\mathcal{H}_s$** is coupled to **the controller, another quantum system**; the composite system of Hilbert space $\boldsymbol{\mathcal{H}_s} \otimes \boldsymbol{\mathcal{H}_c}$, is an open-quantum system relaxing to some target (separable) state. Relaxation behaviors in open quantum systems can be exploited: optical pumping of Alfred Kastler.

[1] Wiseman/Milburn: Quantum Measurement and Control, 2009, Cambridge University Press.

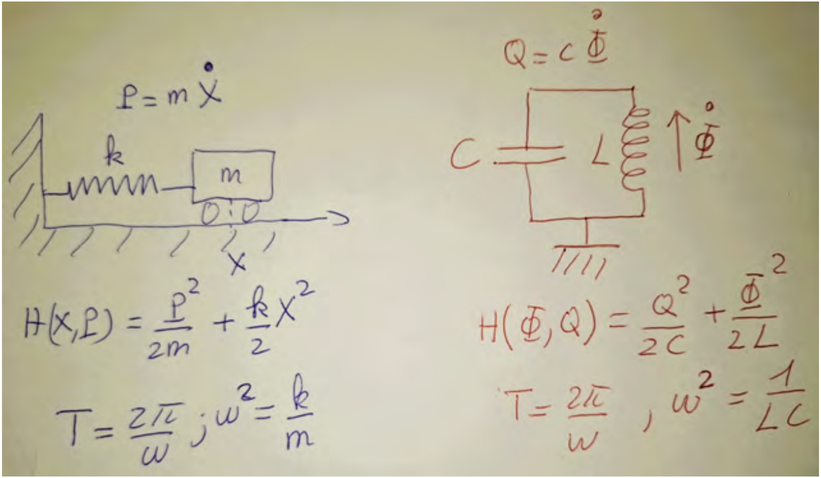
Outline

Linear/nonlinear oscillators

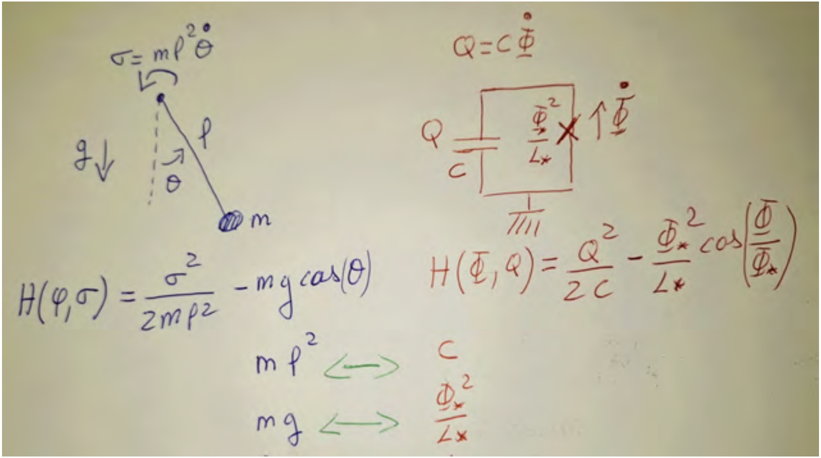
Classical controller

Quantum controller

Linear mechanical/electrical oscillators



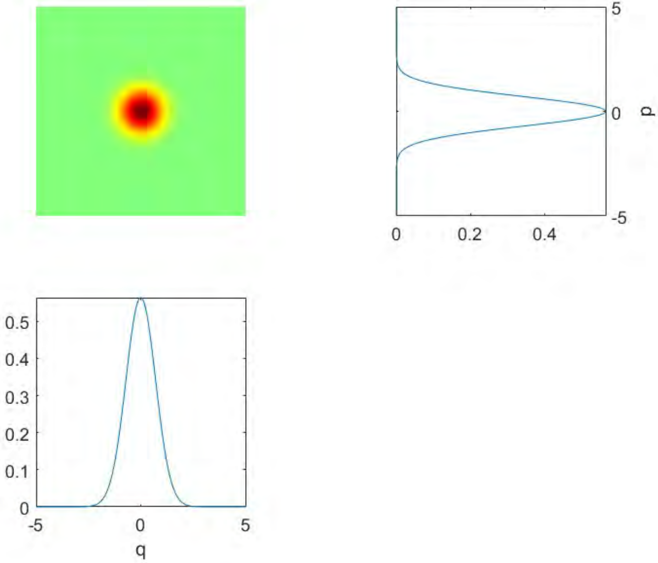
Nonlinear oscillators: pendulum and Josephson circuit



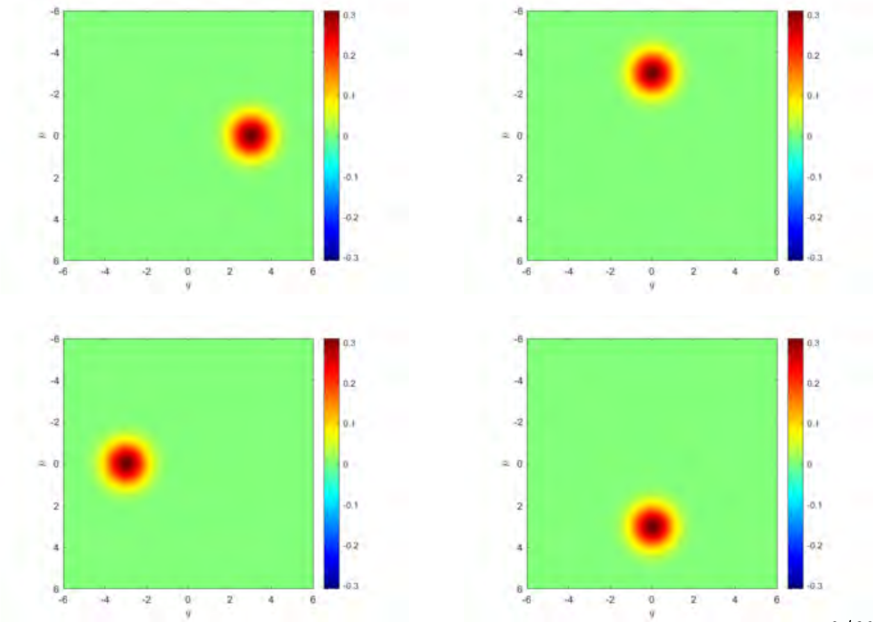
Phase space and Wigner function of a quantum oscillator

Some slides from PhD defense of Léon Carde, November 25, 2025.

Wigner function W of vacuum $|0\rangle$: $\mathbb{R}^2 \ni (q,p) \mapsto W(q,p) \in \left[-\frac{2}{\pi}, \frac{2}{\pi}\right]$

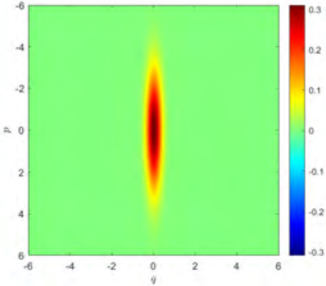


Wigner functions of coherent states $|\alpha\rangle$ with $\alpha = \frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}i, -\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}i$

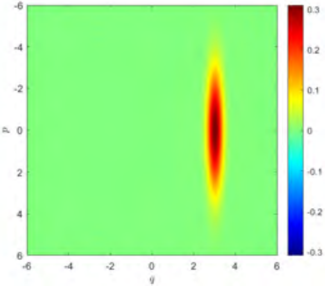


Squeezed coherent states used for gravitational wave detection

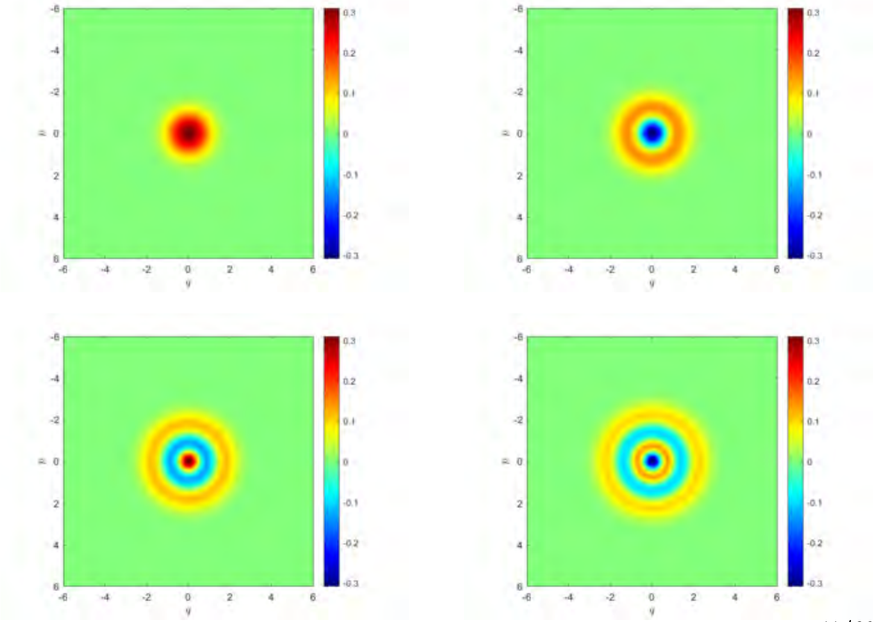
Squeezed vacuum



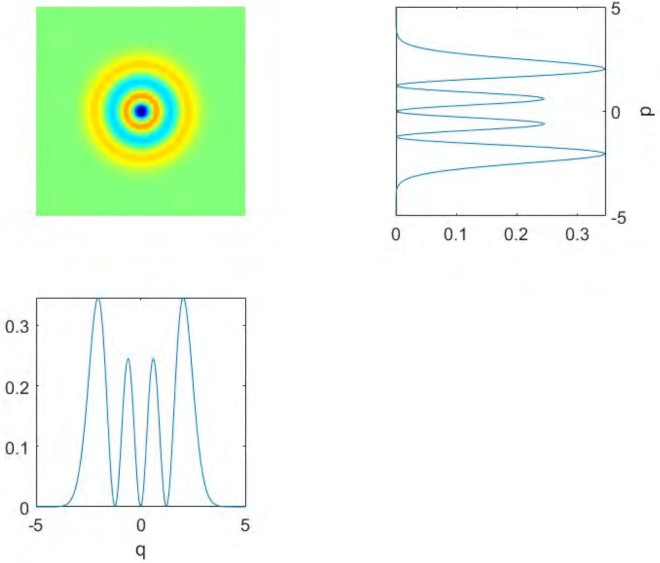
Squeezed coherent state $\alpha = \dfrac{3}{\sqrt{2}}$



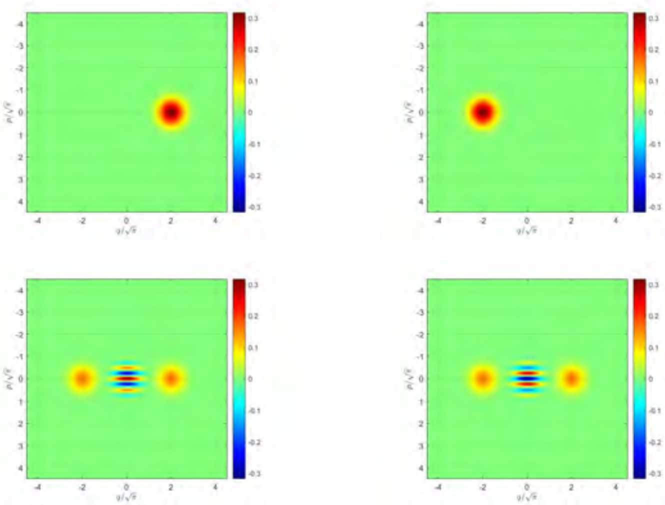
Wigner functions of photon-number states: 0, 1, 2 and 3 photons



Wigner function of 3-photon state $|3\rangle$

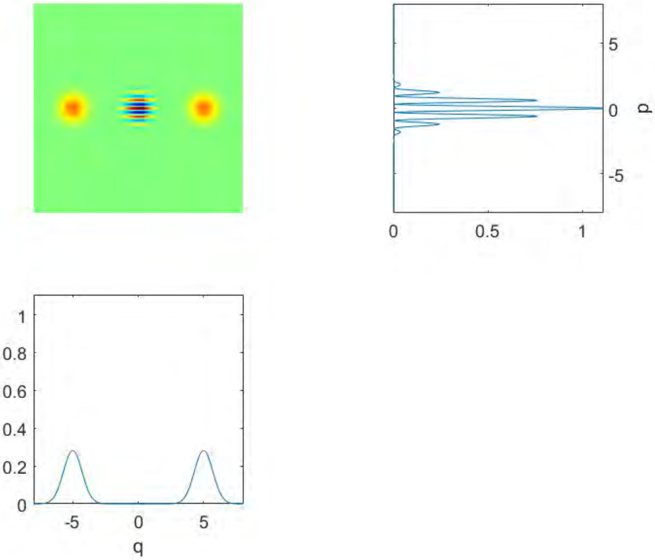


Schrödinger cats for quantum information processing[2]

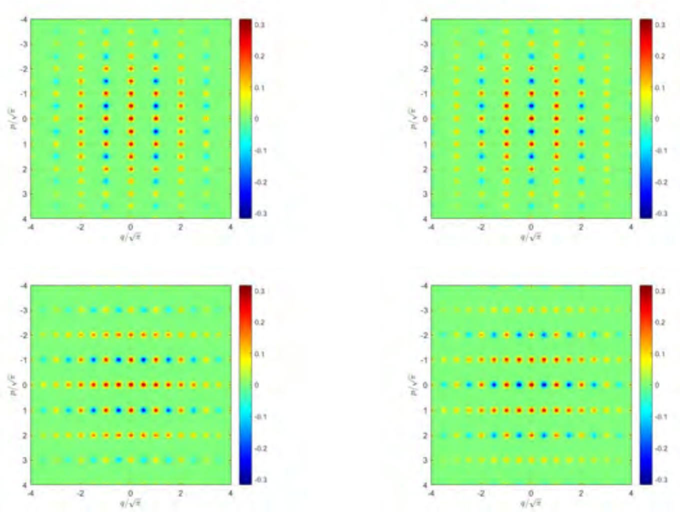


[2] M. Mirrahimi, Z. Leghtas, . . . , **M. Devoret**: Dynamically protected cat-qubits: a new paradigm for universal quantum computation. 2014, New Journal of Physics.

Wigner function of the cat-state $|+_L\rangle$

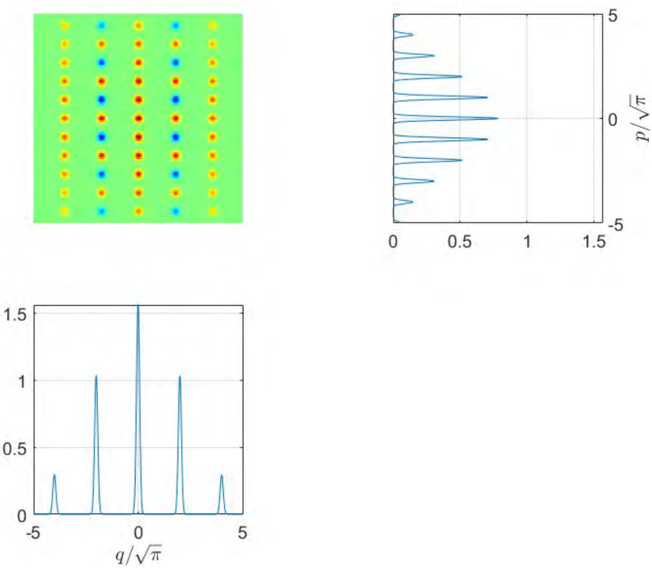


Grid-states for quantum information processing: GKP-qubit[3]



[3] D. Gottesman, A. Kitaev, J. Preskill: Encoding a qubit in an oscillator. PRA, 2001.

Wigner function of the GKP state $|0_L\rangle$



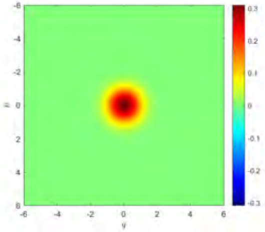
Outline

Linear/nonlinear oscillators

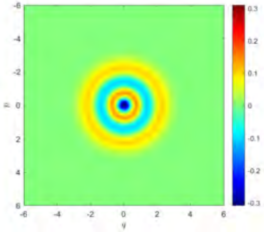
Classical controller

Quantum controller

Stabilization of the 3-photon state with a classical controller



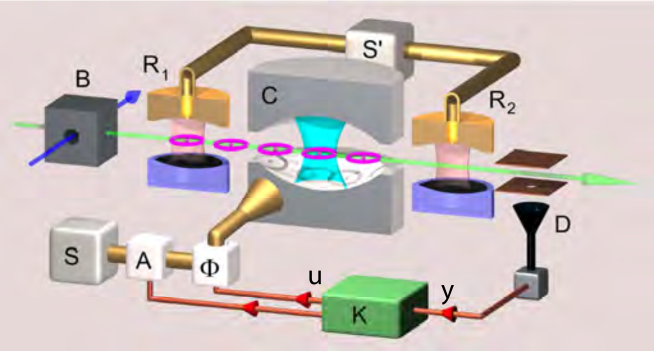
$\longmapsto$



2011: first experimental realization of a **quantum state feedback**[5]

The photon box of the Laboratoire Kastler-Brossel (LKB):
S.Haroche, J.M.Raimond and M. Brune.

[4]



Stabilization of a quantum state with exactly $n = 0, 1, 2, 3, \ldots$ **photon(s).**

[4] Courtesy of Igor Dotsenko. **Sampling period** 80 μs.

[5] C. Sayrin, ..., **S. Haroche**, Nature 477, 73-77, September 2011.

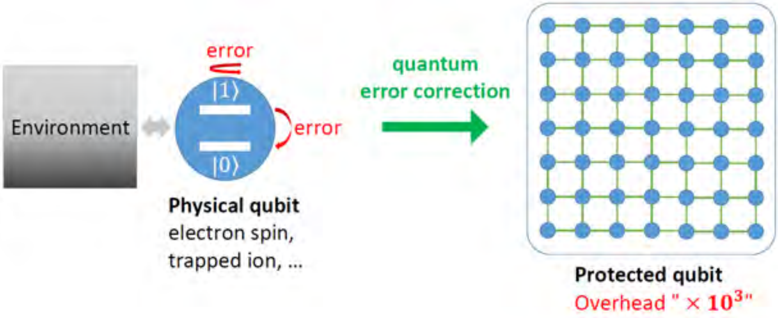
Outline

Quantum Error Correction (QEC): 2D redundancy

It is enough to correct only

1. bit-flip: $\alpha|0\rangle + \beta|1\rangle \mapsto \alpha|1\rangle + \beta|0\rangle$
2. phase-flip: $\alpha|0\rangle + \beta|1\rangle \mapsto -\alpha|0\rangle + \beta|1\rangle$

Key assumption: independence of error probability for each physical qubit (local error).



Some slides from PhD defense of Léon Carde, November 25, 2025.

Bosonic code with cat-qubits

- Quantum error corrrection requires redundancy.
- Bosonic code: instead of encoding a logical qubit in N physical qubits living in $\mathbb{C}^{2^N}$, encode a logical qubit in an harmonic oscillator living in Fock space $\text{span}\{|0\rangle, |1\rangle, \ldots, |n\rangle, \ldots\} \sim L^2(\mathbb{R}, \mathbb{C})$ of infinite dimension.
- Cat-qubit [6]: $|\psi_L\rangle \in \text{span}\{|\alpha\rangle, |\text{-}\alpha\rangle\}$ where $|\alpha\rangle$ is the coherent state of real amplitude α: $\widehat{a}|\alpha\rangle = \alpha|\alpha\rangle$ with $\widehat{a} = (\widehat{q} + i\widehat{p})/\sqrt{2}$ and $[\widehat{q}, \widehat{p}] = i$:

$$|\psi\rangle \sim \psi(q) \in L^2(\mathbb{R}, \mathbb{C}),\ \widehat{q}|\psi\rangle \sim q\psi(q),\ \widehat{p}|\psi\rangle \sim -i\frac{d\psi}{dq}(q),\ |\alpha\rangle \sim \frac{\exp\left(-\frac{(q-\alpha\sqrt{2})^2}{2}\right)}{\sqrt{2\pi}}.$$

- Stabilisation of cat-qubit via a single Lindblad dissipator $\widehat{L} = \widehat{a}^2 - \alpha^2$. For any initial density operator $\rho(0)$, the solution $\rho(t)$ of

$$\frac{d}{dt}\rho = \widehat{L}\rho\widehat{L}^\dagger - \frac{1}{2}(\widehat{L}^\dagger\widehat{L}\rho + \rho\widehat{L}^\dagger\widehat{L})$$

converges exponentially towards a steady-state density operator since

$$\frac{d}{dt}\ \text{Tr}\left(\widehat{L}^\dagger\widehat{L}\rho\right) \leq -2\ \text{Tr}\left(\widehat{L}^\dagger\widehat{L}\rho\right), \quad \ker\widehat{L} = \text{span}\{|\alpha\rangle, |\text{-}\alpha\rangle\}.$$

Any density operator with support in $\text{span}\{|\alpha\rangle, |\text{-}\alpha\rangle\}$ is a steady-state.

[6] M. Mirrahimi, Z. Leghtas, ..., **M. Devoret**: Dynamically protected cat-qubits: a new paradigm for universal quantum computation. 2014, New Journal of Physics.

Asymmetrically Threaded SQUID (ATS) stabilizing a cat-qubit [7]

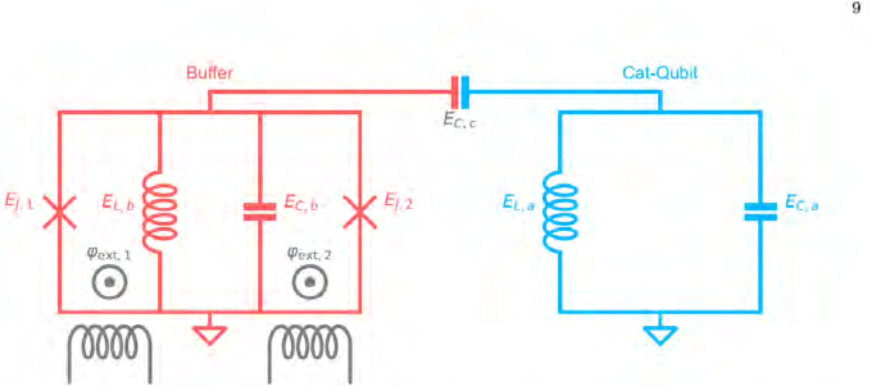
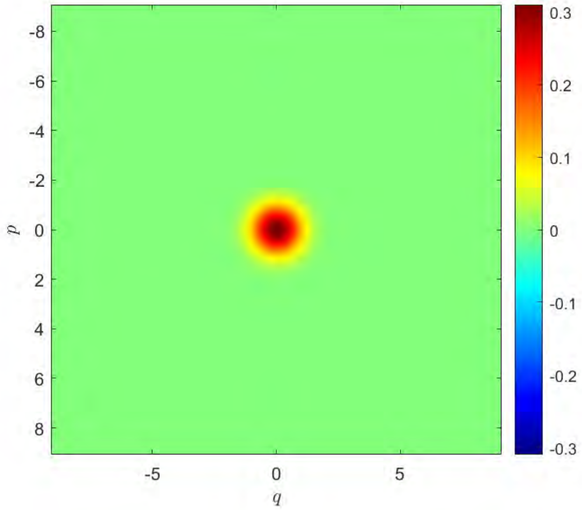


Figure S3. Equivalent circuit diagram. The cat-qubit (blue), a linear resonator, is capacitively coupled to the buffer (red). One recovers the circuit of Fig. 2 by replacing the buffer inductance with a 5-junction array and by setting $\varphi_\Sigma = (\varphi_{ext,1} + \varphi_{ext,2})/2$ and $\varphi_\Delta = (\varphi_{ext,1} - \varphi_{ext,2})/2$. Not shown here: the buffer is capacitively coupled to a transmission line, the cat-qubit resonator is coupled to a transmon qubit

[7] R. Lescanne, . . . , Z. Leghtas: Exponential suppression of bit-flips in a qubit encoded in an oscillator. Nature Physics (2020). U. Reglade, . . . , Z. Leghtas: Quantum control of a cat-qubit with bit-flip times exceeding ten seconds. Nature (2024). Startup Alice&Bob

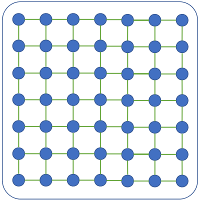
Generation/stabilization/decoherence of the cat-qubit $|+_L\rangle$.



Numerical simulation based on a Lindblad master equation

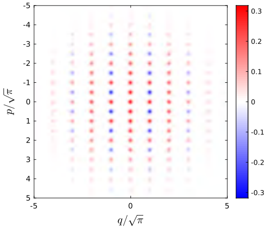
Local noise assumption (1)

DV-QEC



$$\mathcal{H} = \left(\mathbb{C}^2\right)^{\otimes N}$$
Dimension: 2^N

CV-QEC



$$\mathcal{H} = L^2(\mathbb{R}, \mathbb{C})$$
Dimension: $+\infty$

Wave function $|\psi\rangle : \mathbb{R} \ni q \mapsto \psi(q) \in \mathbb{C}$, and Wigner function

$$\mathbb{R}^2 \ni (q,p) \mapsto W^{|\psi\rangle\langle\psi|}(q,p) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \psi^*\left(q - \frac{u}{2}\right) \psi\left(q + \frac{u}{2}\right) e^{-2ipu} du.$$

Local error operators $\hat{q}$ and $\hat{p}$ ($[\hat{q},\hat{p}] = i$) on $|\psi\rangle$: small random shifts along q ($e^{i\pm\epsilon\hat{p}} \equiv e^{\pm\epsilon d/dq}$) and p ($e^{i\pm\epsilon\hat{q}} \equiv e^{\mp\epsilon d/dp}$) similar to diffusion along q and p axis for $W^{|\psi\rangle\langle\psi|}$.

Local noise assumption (2)

For a density operator ρ, its Wigner function

$$\mathbb{R}^2 \ni (q,p) \mapsto W^\rho(q,p) \in \mathbb{R}$$

reads ($\widehat{a} = \frac{\widehat{q}+i\widehat{p}}{\sqrt{2}}$)

$$W^\rho(q,p) = \tfrac{1}{\pi}\ \mathrm{Tr}\left(e^{i\pi \widehat{a}^\dagger \widehat{a}}\ e^{i(p\widehat{q}-q\widehat{p})}\ \rho\ e^{-i(p\widehat{q}-q\widehat{p})} \right)$$

Since $\left(\mathcal{D}_{\widehat{L}}(\rho) = \widehat{L}\rho\widehat{L}^\dagger - (\widehat{L}^\dagger\widehat{L}\rho + \rho\widehat{L}^\dagger\widehat{L})/2\right)$

$$W^{\mathcal{D}_{\widehat{q}}(\rho)} = \frac{1}{2}\frac{\partial^2}{\partial p^2} W^\rho, \quad W^{\mathcal{D}_{\widehat{p}}(\rho)} = \frac{1}{2}\frac{\partial^2}{\partial q^2} W^\rho$$

and

$$W^{\mathcal{D}_{\widehat{a}}(\rho)} = \frac{1}{2}\frac{\partial}{\partial q}(qW^\rho) + \frac{1}{2}\frac{\partial}{\partial p}(pW^\rho) + \frac{1}{2}\frac{\partial^2}{\partial q^2} W^\rho + \frac{1}{2}\frac{\partial^2}{\partial p^2} W^\rho$$

dominant errors on ρ correspond to local differential operators in the phase-space (q,p).

Grid-states and GKP-qubits

- Poisson summation formula: the Fourier transform of Dirac comb $f(q)$ of period T is a Dirac comb $g(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(q) e^{-iqp} dq$ of period $2\pi/T$.

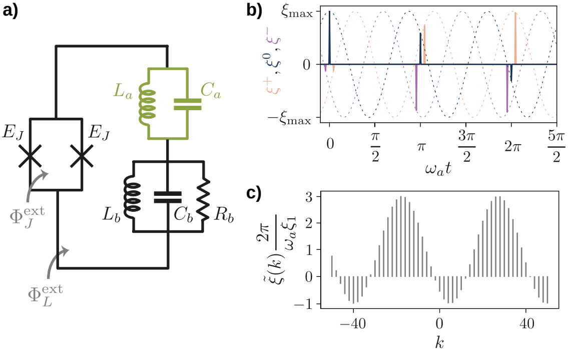
infinite energy grid-states	q representation	p representation
$\|0_L\rangle$	$\sum_k \delta(q - 2k\sqrt{\pi})$	$\sum_k \delta(p - k\sqrt{\pi})$
$\|1_L\rangle$	$\sum_k \delta(q - 2(k+1)\sqrt{\pi})$	$\sum_k (-1)^k \delta(p - k\sqrt{\pi})$
$\|+_L\rangle \sim \|0_L\rangle + \|1_L\rangle$	$\sum_k \delta(q - k)\sqrt{\pi})$	$\sum_k \delta(p - 2k\sqrt{\pi})$
$\|-_L\rangle \sim \|0_L\rangle - \|1_L\rangle$	$\sum_k (-1)^k \delta(q - k)\sqrt{\pi})$	$\sum_k \delta(p - 2(k+1)\sqrt{\pi})$

- Pauli operators of a GKP-qubit[8] with Bloch coordinates $(x, y, z) \in \mathbb{R}^3$:
$$\widehat{Z} = \text{sign}(\cos(\sqrt{\pi}\widehat{q})),\ \widehat{X} = \text{sign}(\cos(\sqrt{\pi}\widehat{p})) \text{ and } \widehat{Y} = -i\widehat{Z}\widehat{X}.$$
- 4 stabilizer operators $\widehat{S}$ relying on commuting modular operators in $\widehat{q}$ and $\widehat{p}$:
$$\forall \widehat{S} \in \{e^{i2\sqrt{\pi}\widehat{q}}, e^{i2\sqrt{\pi}\widehat{p}}, e^{-i2\sqrt{\pi}\widehat{q}}, e^{-i2\sqrt{\pi}\widehat{p}}\} \text{ and } \forall |\psi_L\rangle \in \text{span}\{|0_L\rangle, |1_L\rangle\}:\ \widehat{S}|\psi_L\rangle = |\psi_L\rangle$$
- Finite energy regularization with $0 < \epsilon \ll 1$,
$$|0_\epsilon\rangle \approx e^{-\epsilon \widehat{a}^\dagger \widehat{a}} |0_L\rangle, \quad |1_\epsilon\rangle \approx e^{-\epsilon \widehat{a}^\dagger \widehat{a}} |0_L\rangle,$$
where $\widehat{a}^\dagger \widehat{a} = \frac{1}{2}(\widehat{q}^2 + \widehat{p}^2) \sim \frac{1}{2}(q^2 + \partial^2/\partial q^2)$, provides a finite-energy code space where **any small local error can be corrected**[9]**.**

[8] D. Gottesman, A. Kitaev and J. Preskill: Encoding a qubit in an oscillator. Physical Review A, 2001.

[9] 3 recent experiments stabilizing GKP-qubits via classical controllers: Ph. Campagne-Ibarcq et al. "Quantum error correction of a qubit encoded in grid states of an oscillator" Nature (2020); B. de Neeve et al. "Error correction of a logical grid state qubit by dissipative pumping" Nature (2022); V. Sivak et al. "Real-Time Quantum Error Correction beyond Break-Even" Nature (2023).

Engineering modular dissipation with super-conducting Josephson circuits [10]



High impedance $\sqrt{L_a/C_a}$ and low pulsation $1/\sqrt{L_a C_a}$ for storage mode $\widehat{a}$.

High pulsation $1/\sqrt{L_b C_b}$ of damped mode $\widehat{b}$ (quantum controller $R_b > 0$).

Josephson energy E_J between $\hbar/\sqrt{L_a C_a}$ and $\hbar/\sqrt{L_b C_b}$.

Classical open-loop control signals $\Phi_J^{ext}(t)$ and $\Phi_L^{ext}(t)$ made of short pulses.

Mathematical analysis to recover master equation with dissipators $\widehat{L}_k$.

Numerical simulations to test robustness versus experimental imperfections.

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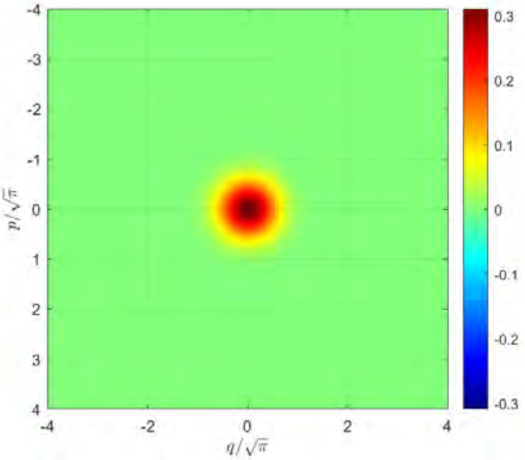
[10] L.A. Sellem, A. Salette, Z. Leghtas, M. Mirrahimi, PR, Ph. Campagne-Ibarcq: A GKP qubit protected by dissipation in a high-impedance superconducting circuit driven by a microwave frequency comb. PRX 2025 (arXiv:2304.01425).

Generation/stabilization/decoherence of magic state $\sin\left(\frac{\pi}{8}\right)|0_L\rangle + \cos\left(\frac{\pi}{8}\right)|1_L\rangle$

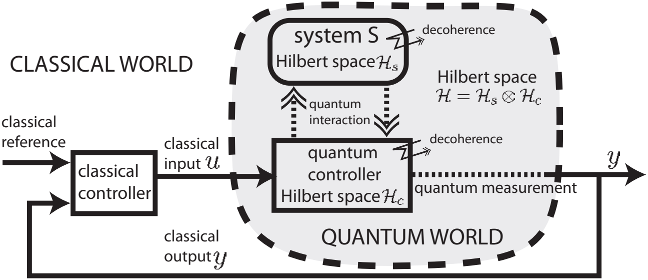


Numerical simulation relying a Lindbald master equation[11].

[11] L.A. Sellem, A. Salette, Z. Leghtas, M. Mirrahimi, PR, Ph. Campagne-Ibarcq: A GKP qubit protected by dissipation in a high-impedance superconducting circuit driven by a microwave frequency comb. PRX 2025 (arXiv:2304.01425).

Concluding remark

Quantum feedback engineering for robust quantum information processing



To protect quantum information stored in system S:

- fast stabilization and protection mainly achieved by **quantum controllers** (autonomous feedback stabilizing decoherence-free sub-spaces);
- slow decoherence and perturbations, parameter estimation mainly tackled by **classical controllers and estimation algorithms** (measurement-based feedback and parameter estimation "finishing the job")

Need of adapted mathematical and numerical methods for high-precision dynamical modeling and control based on (stochastic) master equations.

Quantic research group ENS/Inria/Mines/CNRS, June 2023

Modeling based on three quantum features[12]

1. Schrödinger: wave funct. $|\psi\rangle \in \mathcal{H}$ or density op. $\rho \sim |\psi\rangle\langle\psi|$

$$\frac{d}{dt}|\psi\rangle = -\tfrac{i}{\hbar}\widehat{H}|\psi\rangle, \quad \frac{d}{dt}\rho = -\tfrac{i}{\hbar}[\widehat{H}, \rho], \quad \widehat{H} = \widehat{H}_0 + u\widehat{H}_1$$

2. Origin of dissipation: collapse of the wave packet induced by the measurement of observable $\widehat{O}$ with spectral decomp. $\sum_\mu \lambda_\mu \widehat{P}_\mu$:
   - measurement outcome μ with proba. $\mathbb{P}_\mu = \langle\psi|\widehat{P}_\mu|\psi\rangle = \text{Tr}\left(\rho\widehat{P}_\mu\right)$ depending on $|\psi\rangle$, ρ just before the measurement
   - measurement back-action if outcome $\mu = y$:

$$|\psi\rangle \mapsto |\psi\rangle_+ = \frac{\widehat{P}_y|\psi\rangle}{\sqrt{\langle\psi|\widehat{P}_y|\psi\rangle}}, \quad \rho \mapsto \rho_+ = \frac{\widehat{P}_y\rho\widehat{P}_y}{\text{Tr}\left(\rho\widehat{P}_y\right)}$$

3. Tensor product for the description of composite systems (S, M):
   - Hilbert space $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_M$
   - Hamiltonian $\widehat{H} = \widehat{H}_S \otimes \widehat{I}_M + \widehat{H}_{int} + \widehat{I}_S \otimes \widehat{H}_M$
   - observable on sub-system M only: $\widehat{O} = \widehat{I}_S \otimes \boldsymbol{O}_M$.

[12] See books: Davies in 1976; Haroche/Raimond in 2006; Wiseman/Milburn in 2009; Gardiner/Zoller in 2014/2015; ...

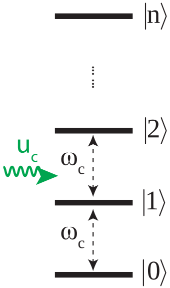
Quantum harmonic oscillator (spring system) [13]

- Hilbert space:
  $\mathcal{H}_S = \left\{ \sum_{n \geq 0} \psi_n |n\rangle, \ (\psi_n)_{n \geq 0} \in l^2(\mathbb{C}) \right\} \equiv L^2(\mathbb{R}, \mathbb{C})$
- Quantum state space:
  $\mathcal{D} = \{\rho \in \mathcal{L}(\mathcal{H}_S), \rho^\dagger = \rho, \ \mathrm{Tr}\,(\rho) = 1, \rho \geq 0\}$.
- Operators and commutations:
  $\widehat{a}|n\rangle = \sqrt{n}\ |n\text{-}1\rangle$, $\widehat{a}^\dagger |n\rangle = \sqrt{n+1}|n+1\rangle$;
  $\widehat{N} = \widehat{a}^\dagger \widehat{a}$, $\widehat{N}|n\rangle = n|n\rangle$;
  $[\widehat{a}, \widehat{a}^\dagger] = \widehat{I}$, $\widehat{a} f(\widehat{N}) = f(\widehat{N} + \widehat{I})\widehat{a}$;
  $\widehat{D}_\alpha = e^{\alpha \widehat{a}^\dagger - \alpha^\dagger \widehat{a}}$.
  $\widehat{a} = \frac{\widehat{Q} + i\widehat{P}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \left(q + \frac{\partial}{\partial q} \right)$, $[\widehat{Q}, \widehat{P}] = \imath \widehat{I}$.
- Hamiltonian: $\widehat{H}_S = \omega_c \widehat{a}^\dagger \widehat{a} + u_c(\widehat{a} + \widehat{a}^\dagger)$.
  (associated classical dynamics: $\frac{dq}{dt} = \omega_c p$, $\frac{dp}{dt} = -\omega_c q - u_c$).
- Classical pure state $\equiv$ coherent state $|\alpha\rangle$

  $$\alpha \in \mathbb{C} : \ |\alpha\rangle = \sum_{n \geq 0} \left(e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} \right) |n\rangle; \ |\alpha\rangle \equiv \frac{1}{\pi^{1/4}} e^{\imath \sqrt{2} q \Im \alpha} e^{-\frac{(q - \sqrt{2} \Re \alpha)^2}{2}}$$

  $\widehat{a}|\alpha\rangle = \alpha|\alpha\rangle$, $\widehat{D}_\alpha|0\rangle = |\alpha\rangle$.
- Wigner function of a density operator ρ:

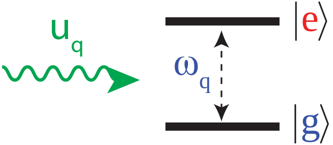
  $$\mathbb{C} \ni \alpha = \frac{q+ip}{\sqrt{2}} \mapsto W^\rho(q,p) = \ \mathrm{Tr}\left(e^{i\pi \widehat{N}} \ \widehat{D}_\alpha \ \rho \ \widehat{D}_{-\alpha} \right)$$



[13] See S. M. Barnett, P.M. Radmore (2003): Methods in Theoretical Quantum Optics. Oxford University Press.

Qubit (2-level system, half-spin) [14]

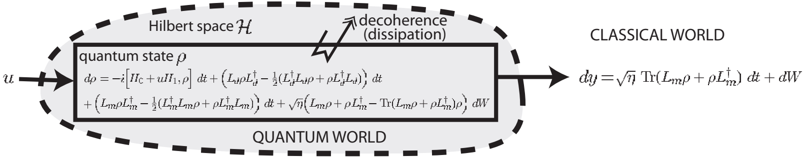
- Hilbert space:
  $\mathcal{H}_M = \mathbb{C}^2 = \left\{ c_g |g\rangle + c_e |e\rangle,\ c_g, c_e \in \mathbb{C} \right\}.$
- Quantum state space:
  $\mathcal{D} = \{\rho \in \mathcal{L}(\mathcal{H}_M), \rho^\dagger = \rho,\ \mathrm{Tr}\,(\rho) = 1, \rho \geq 0\}$.
- Operators and commutations:
  $\widehat{\sigma}_- = |g\rangle\langle e|,\ \widehat{\sigma}_+ = \widehat{\sigma}_-^\dagger = |e\rangle\langle g|$
  $\widehat{X} \equiv \widehat{\sigma}_x = \widehat{\sigma}_- + \widehat{\sigma}_+ = |g\rangle\langle e| + |e\rangle\langle g|;$
  $\widehat{Y} \equiv \widehat{\sigma}_y = i\widehat{\sigma}_- - i\widehat{\sigma}_+ = i|g\rangle\langle e| - i|e\rangle\langle g|;$
  $\widehat{Z} \equiv \widehat{\sigma}_z = \widehat{\sigma}_+\widehat{\sigma}_- - \widehat{\sigma}_-\widehat{\sigma}_+ = |e\rangle\langle e| - |g\rangle\langle g|;$
  $\widehat{\sigma}_x^2 = \widehat{I},\ \widehat{\sigma}_x\widehat{\sigma}_y = i\widehat{\sigma}_z,\ [\widehat{\sigma}_x, \widehat{\sigma}_y] = 2i\widehat{\sigma}_z,\ \ldots$



- Hamiltonian: $\mathrm{H}_M = \omega_q \widehat{\sigma}_z / 2 + \mathrm{u_q} \widehat{\sigma}_x$.
- Bloch sphere representation:
  $\mathcal{D} = \left\{ \frac{1}{2}\left(\widehat{I} + x\widehat{\sigma}_x + y\widehat{\sigma}_y + z\widehat{\sigma}_z\right) \mid (x, y, z) \in \mathbb{R}^3,\ x^2 + y^2 + z^2 \leq 1 \right\}$

[14] See S. M. Barnett, P.M. Radmore (2003): Methods in Theoretical Quantum Optics. Oxford University Press.

Continuous-time dynamics based on diffusive Stochastic Master Equation (SME) [15]



Continuous-time models: stochastic differential systems (Itō formulation)
Control input $\boldsymbol{u}$, state ρ (density op.), measured output $\boldsymbol{y}$:

$$d\rho_t = \Big(- i[\widehat{H}_0 + \boldsymbol{u_t}\widehat{H}_1, \rho_t] + \sum_{\nu=d,m} \widehat{L}_\nu \rho_t \widehat{L}_\nu^\dagger - \frac{1}{2}(\widehat{L}_\nu^\dagger \widehat{L}_\nu \rho_t + \rho_t \widehat{L}_\nu^\dagger \widehat{L}_\nu)\Big) dt + \sqrt{\eta_m}\Big(\widehat{L}_m\rho_t + \rho_t\widehat{L}_m^\dagger - \ \mathrm{Tr}\left((\widehat{L}_m + \widehat{L}_m^\dagger)\rho_t\right)\rho_t\Big) \boldsymbol{dW_t}$$

driven by the Wiener process $\boldsymbol{W_t}$, with measurement $\boldsymbol{y_t}$,

$$\boldsymbol{dy_t} = \sqrt{\eta_m}\ \mathrm{Tr}\left((\widehat{L}_m + \widehat{L}_m^\dagger)\,\rho_t\right)\ dt + \boldsymbol{dW_t} \quad \text{detection efficiencies } \eta_m \in [0,1].$$

Measurement backaction: $d\rho_t$ **and** $\boldsymbol{dy_t}$ **share the same noises** $\boldsymbol{dW_t}$. Very different from Kalman I/O state-space description used in control engineering.

[15] A. Barchielli, M. Gregoratti (2009): Quantum Trajectories and Measurements in Continuous Time: the Diffusive Case. Springer Verlag.