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G-score, the Maximum Cardinality Matching G_n series as a benchmark for quantum computers

Benchmark for quantum optimization heuristics

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Section 1

Context

French QC applicative benchmarking “BACQ”

A polyvalent suite of benchmark for applications of QC addressing several fields

Simulation of Quantum Physics
(Quantum phase transition: Ising model in transverse field, 2D Hubbard model)

Optimization
(including Q-Score/MAXCUT)

Linear Systems Solving

Prime factorization

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We aim to address all possible kind of QC harware, including:

- gate-based quantum computers
- analog quantum computers and quantum annealers
- work on "quantum inspired" solutions (e.g. NEC, Fujitsu ...) as reference points

The quantum utility gap & its evaluation

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- N.B.: such approaches encompasses the capabilities of the hardware and the software tools
- But this is what the end-user cares about, so it is fine

Section 2

The Gn series, a suite of constrained optimization problems

The Gn series: a hard problem for optimization heuristics

G_n series of optimization problem:

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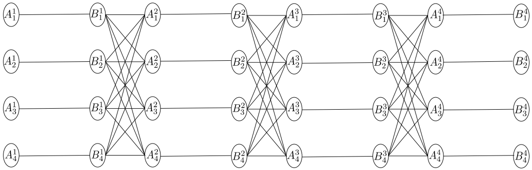

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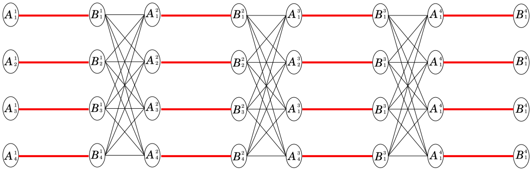


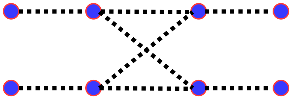
The Gn series on QA: from hard constraints to soft ones

QUBO (for minimazation):

$$q_{ee} = -1 - 2\lambda \text{ et } q_{ee'} = \begin{cases} 2\lambda & \text{si } \exists v \in N/e \in \Gamma(v) \text{ and } e' \in \Gamma(v) \\ 0 & \textit{otherwise} \end{cases}$$

As $\sum_{e \in E} x_e \leq card\{E\}$ we can decide for $\lambda = card\{E\} = (n+1)^3$ as an upper value [3]

- Example: G1



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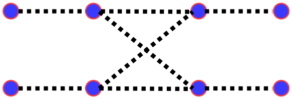

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- When n grows, the number of edge per vertex increases linearly
- $\Rightarrow$ the minor-embedding is rapidly an issue

Why this series of problem is interesting as a benchmark

At least a handful of papers have studied the case of G_n on various DWave QPUs and beyond:

- 2021 Initial study on DWave-2000Q [Vert et al]
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- MCM G_n is an imperfect fit for QUBO approaches, because of hard constraints

Section 3

From a series of problem to a benchmark score

From experimental results to benchmark scores

How to evaluate the results in a platform-independent way?

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- Differentiating QPU results from Random results:
  - Applying a threshold to probability with statistical significance above randomness
  - For low probability results ($p \leq 10^{-4}$) the threshold is given at a significant 1% threshold (i.e. it is required to have at least 1000 non-filtrated samples)
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In this case we decided on a mix of both Hamming distance and number of violation

- The final score = evaluation of the size of a solvable problem

Definition of G-score and weighted G- score

- The relative Hamming distance to the optimal solution

$$r_H = \frac{d_H}{(n+1)^3} \tag{1}$$

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$$r_d = \left(0, \frac{\mathcal{L} - f}{(n+1)^2}\right) \tag{2}$$

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- We define a ratio of optimality for the first "failed" G_n

$$r_o = \frac{1}{2}(1 - r_H + r_d) \tag{3}$$

Section 4

Definition of G-Score and Experimental results on DWave family of QPUs

G-score results

We define a weighted-score based on the probability of finding the optimal on the last "non-failed" G_n

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Experimental results

QPU architecture	n_s	r_o	p_o	S_G
DWave-2000Q (Chimera)	2	0.9375	84.9%	**54.7**
DWave Advantage (Pegasus)	3	0.904	1.8%	**64.8**
DWave Advantage-2-proto (Zephir)	3	0.936	11.0%	**69.1**
SA	5	0.92	4%	220
VA2 (NEC)	7	0.93	0.7%	513
VA3 (NEC)	7	0.95	100	717

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Classical computing is still significantly better than current results of QA on the latest QPU, but QPUs are getting better

Main points and future work

- Next year, the BACQ consortium expects to provide Benchmark scores on a small subset of current QCs
  - Methodology, code and instances will be free to use to improve on the scores
  - Methods to implement and port to other QPU will be provided in preliminary format
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Future work:

- Porting to other analog QCs:
  - Work should start in 2026-Q1 for Pasqal QC (Ruby) and Quandela QC (Lumy)
  - End Q4-2025 to Q3-2026 for first tests on IBM QPUs
- New optimization benchmark on non quadratic problems (harder for annealers)

Acknowledgments and references

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Merci

Thanks!

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