Application of Multi-Criteria Decision Aiding to Quantum Benchmarking: Some Results on Scale Invariance

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Quantum Benchmarking

> **Aim: assess the performance of implementations of quantum computers**

> **Associated to KPIs (Key Performance Indicators)**

> **Levels of benchmarking**

Level	Scope	Example of KPIs
Component – level	hardware implementation of individual components	• Gate fidelity • channel fidelity
System – level	hardware performance of entire implementations	• quantum volume • layer fidelity
Application – level	User perspective • willing to solve a concrete problem • willing to compare the performance on various hardware (QC, HPC,...)	Q-score

Examples of KPIs

> Simulation of Quantum Physical Models

- Q-Score Many-Body
- Many-Body Fidelity

> Optimization

- Q-Score of « Max-Cut »
- Q-Score of « Max-Clique »
- Max size of « Max Cardinality Matching »
- Accuracy indexed by noise level
- Compilation-dependant criteria
- Probability of obtaining the optimum
- Min case/Max case Gap wrt Optimum
- Size of problem in number of variables, number of Qbits to solve the problem, Class of problem addressed.

Linear systems Solving

- Accuracy indexed by noise level
- Compilation-dependant criteria
- Probability of solving the Problem, Number of variables and precision in Qbits.
- Algorithm-dependant Energetic criteria

Factorization

- Probability to factorize
- Size in Qubit of the calculated number

Generic

- Computation Time
- Latency
- Throughput

Multi-Criteria Decision Aiding (MCDA) for Quantum Benchmarking

> How to combine several KPIs in order to

- Assess globally each solution, or
- Compare several solutions to identify the most preferred one?

> Use of Multi-Criterion Decision Aiding (MCDA)

- Solution (Quantum Computer): $\mathbf{x} = \underbrace{(x_1, \dots, x_n)}_{\text{Values on the KPIs}}$

- We seek to construct the preference $\succcurlyeq$ of stakeholders on solutions

$\mathbf{x} \succcurlyeq \mathbf{y}$ if $\mathbf{x}$ is preferred to $\mathbf{y}$

Notion of criterion

Notion of « criterion »: there exists a preference $\succcurlyeq_i$ on KPI i s.t.

$x_i \succcurlyeq_i y_i$ if x_i is preferred to y_i on KPI i

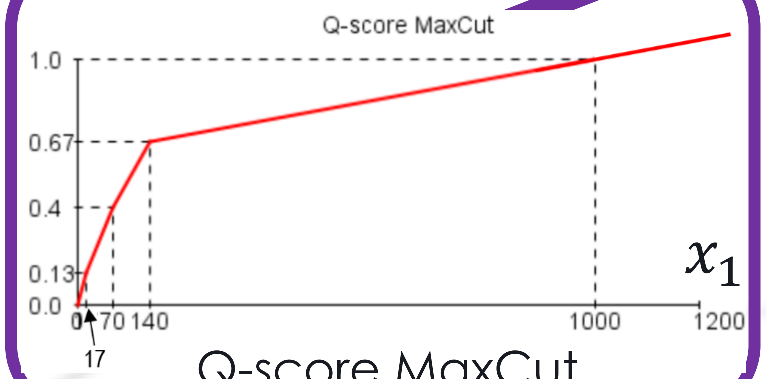
	Q-Score MaxCut	Q-Score MaxClique	Largest size of MCM
	Maximize	Maximize	Maximize
x	145	40	25
y	70	20	10
	$145 \succcurlyeq_1 70$	$40 \succcurlyeq_2 20$	$25 \succcurlyeq_3 10$

MCDA model

> Aggregation Function

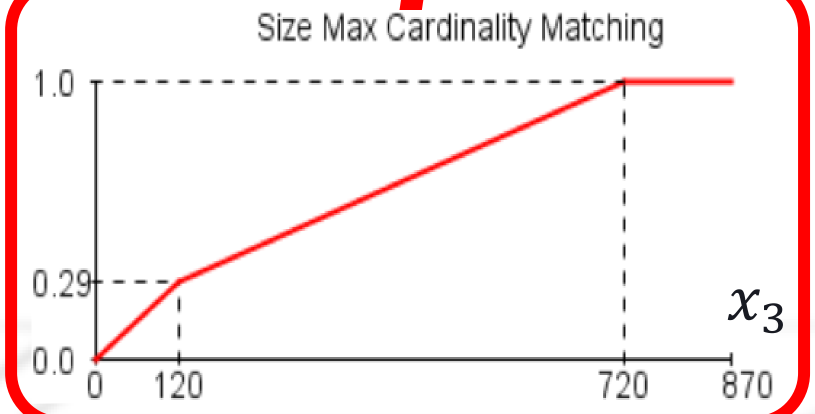
- Weighted Sum
- Geometric Average
- …

$$u(x) = \boxed{H}\left(\boxed{u_1(x_1)}, \boxed{u_2(x_2)}, \boxed{u_3(x_3)}\right)$$



Q-score MaxCut

Q-score MaxClique



Normalization of the KPIs: type of scale of u_i

> Normalization u_i is not uniquely specified:
> "if u_i is admissible, then $\phi_i \circ u_i$ is also admissible"

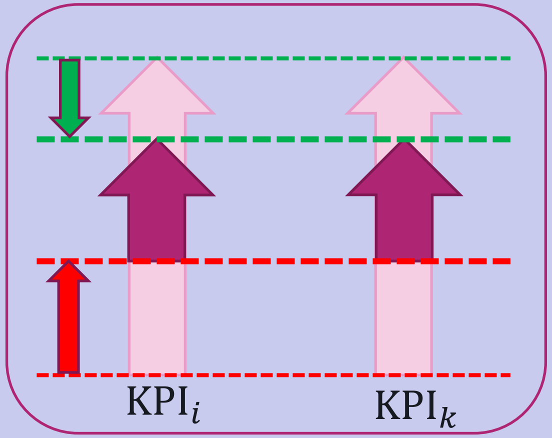
Type of scale	Input preference data	Family of ϕ	Consequence
Ordinal Scale	• Examples of preference $x_i \succcurlyeq_i y_i$	Any strictly increasing function ϕ_i	Too Weak !
Interval Scale	• Examples of preference $x_i \succcurlyeq_i y_i$ • Intensity of preferences: $[x_i \to y_i] \in \{very\ weak, \dots, extreme\}$	Any positive affine transformation $\phi_i(a) = \alpha_i\, a + \beta_i$	Perfect when normalization are combined arithmetically (weighted sum,…)

Normalization of the KPIs: different assumptions on commensurateness

Strong commensurateness

Definition

- All ϕ_i are **similar**
  $\phi_1(a) = \cdots = \phi_n(a) = \alpha\, a + \beta$

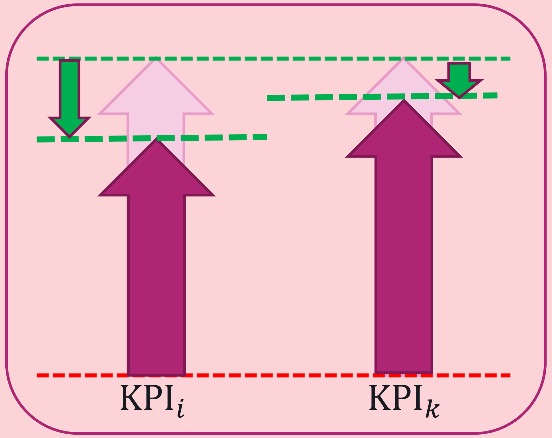


$$\phi_i(a) = 0.3\, a + 0.4$$
$$\phi_k(a) = 0.3\, a + 0.4$$

Weak commensurateness

Definition

- All ϕ_i are **similar** up to a dilation: $\phi_k(u_k) = \alpha_k\, u_k$

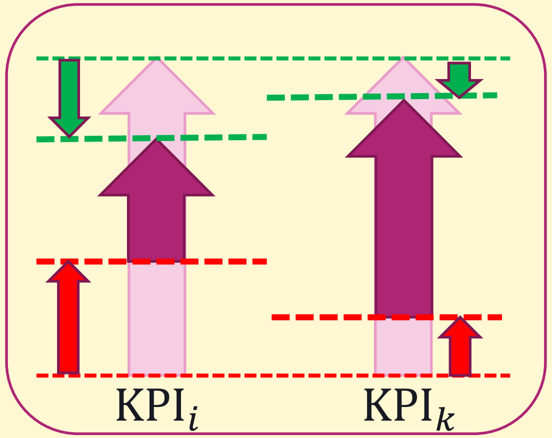


$$\phi_i(a) = 0.8\, a$$
$$\phi_k(a) = 0.9\, a$$

No commensurateness

Definition

- The $u_i's$ can move **independently** on all KPIs:
  $\phi_1(a) = \alpha_1\, a + \beta_1, \ldots,$
  $\phi_n(a) = \alpha_n\, a + \beta_n$



$$\phi_i(a) = 0.3\, a + 0.4$$
$$\phi_k(a) = 0.7\, a + 0.2$$

Normalization of the KPIs: different assumptions on commensurateness

Strong commensurateness

Definition

- All ϕ_i are **similar**
  $\phi_1(a) = \cdots = \phi_n(a) = \alpha\, a + \beta$

Reference levels

- Introduce **2 reference levels**:
  - $\mathfrak{G}_k$: Good level
  - $\mathfrak{B}_k$: Bad level

KPI	$\mathfrak{B}_k$	$\mathfrak{G}_k$
Q-Score Max cut	0	1000
Q-Score Max Clique	0	1000
Largest size of MCM	0	723

Weak commensurateness

Definition

- All ϕ_i are **similar** up to a dilation: $\phi_k(u_k) = \alpha_k\, u_k$

Reference levels

- Introduce **1 reference level**:
  - $\mathfrak{B}_k$: Bad level
- $\mathfrak{B}_k$ =0 for many performance KPIs:
  - Q-Score
  - Largest size of MCM

No commensurateness

Definition

- The $u_i's$ can move **independently** on all KPIs:
  $\phi_1(a) = \alpha_1\, a + \beta_1, \ldots,$
  $\phi_n(a) = \alpha_n\, a + \beta_n$

Reference levels

- No constraint among the $\phi_i's$
- No need for reference levels !

Normalization of the KPIs: different assumptions on commensurateness

Strong commensurateness

Definition

- All ϕ_i are **similar** $\phi_1(a) = \cdots = \phi_n(a) = \alpha\, a + \beta$
- Introduce **2 reference levels**:
  - $\mathfrak{G}_k$: Good level
  - $\mathfrak{B}_k$: Bad level

Consequence on F

Many F are OK: weighted sum, Choquet integral, multi-linear model,…

Weak commensurateness

Definition

- All ϕ_i are **similar** up to a dilation: $\phi_k(u_k) = \alpha_k\, u_k$
- Introduce **1 reference level**:
  - $\mathfrak{B}_k$: Bad level

Consequence on F

Only one model:

$$F(\mathbf{u}) = f(u_1^{w_1} \times \cdots \times u_n^{w_n})$$

No commensurateness

Definition

- The $u_i's$ can move **independently** on all KPIs: $\phi_1(a) = \alpha_1\, a + \beta_1, \ldots, \phi_n(a) = \alpha_n\, a + \beta_n$
- No constraint among the $\phi_i's$

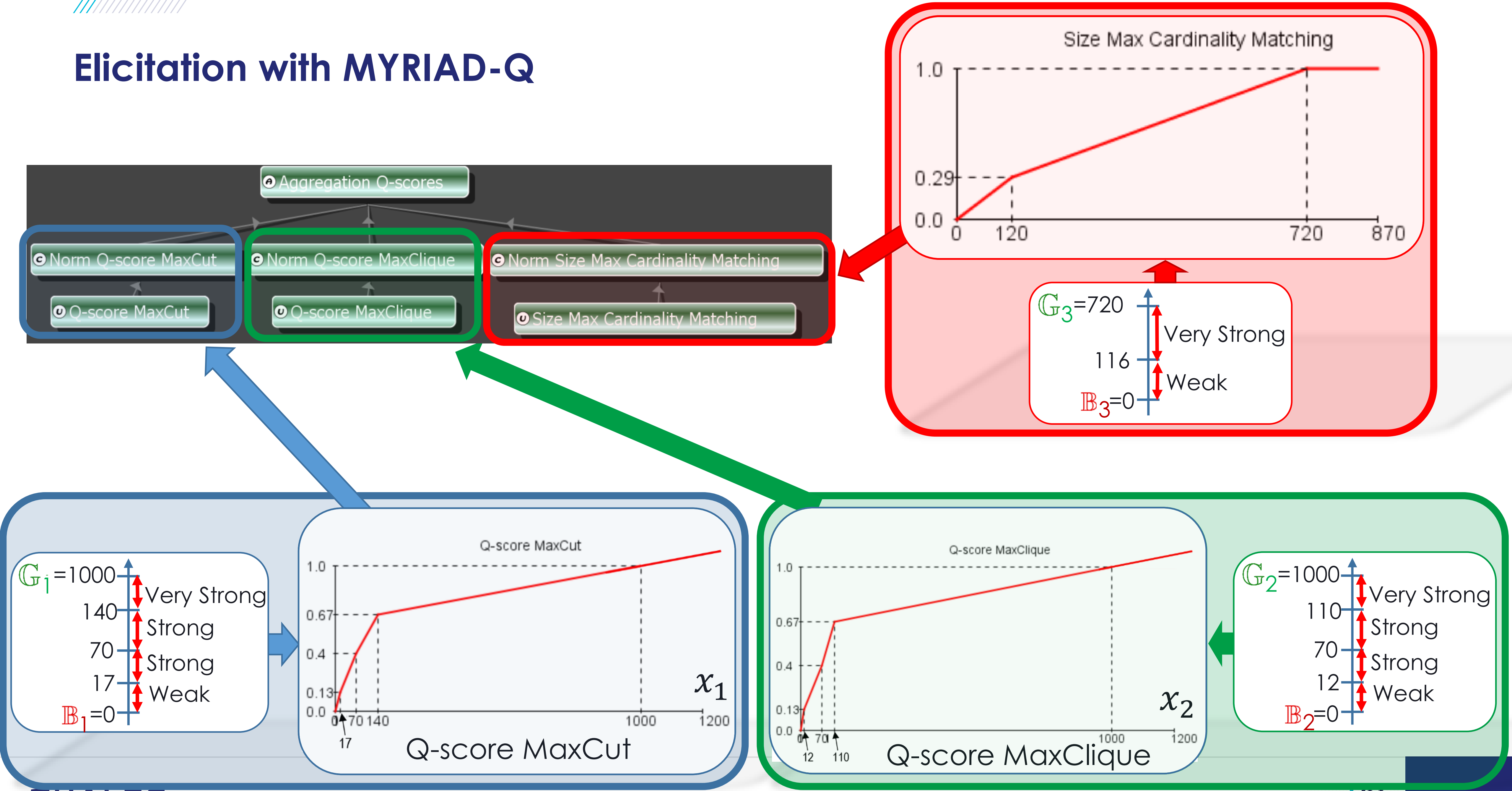
Consequence on F

There is no F !

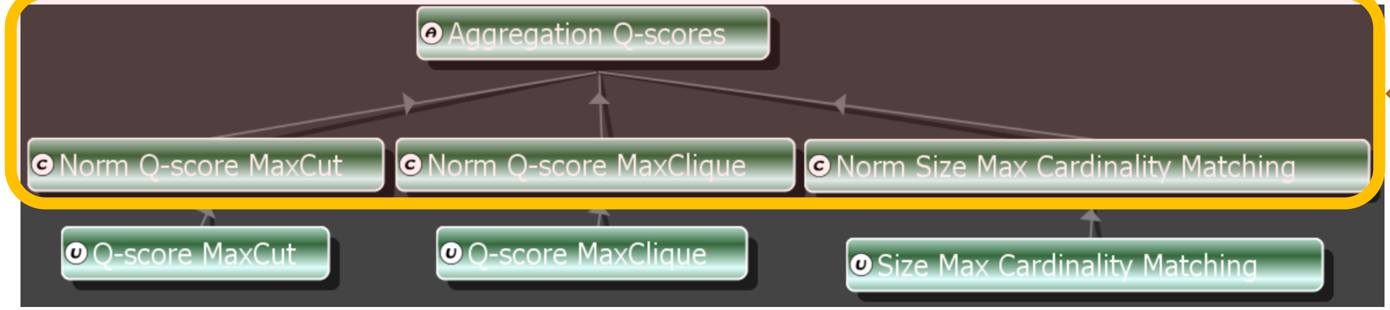
Invariance problem

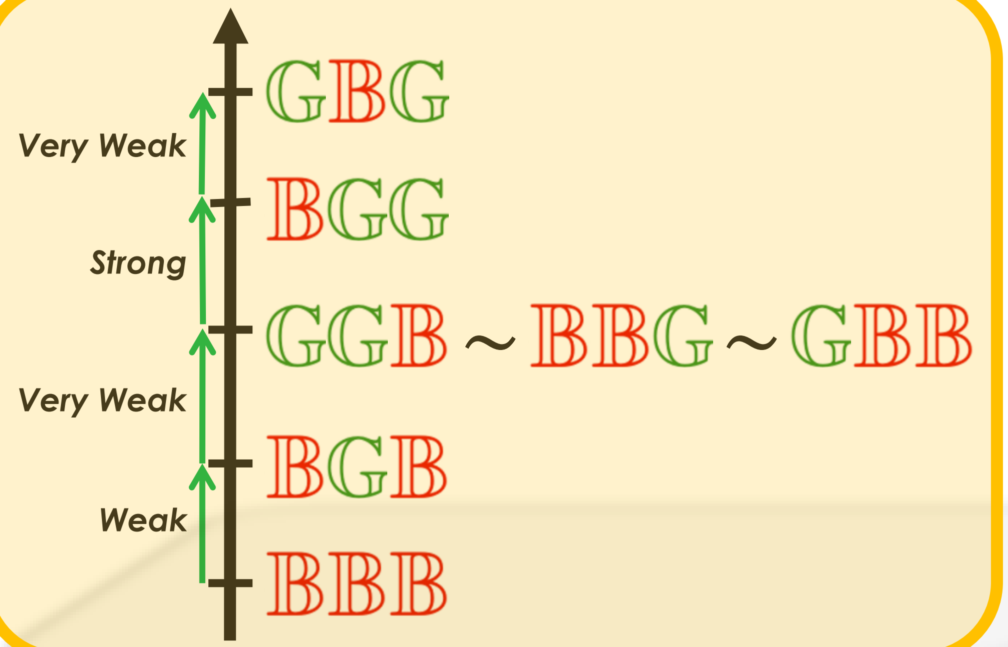
Which aggregation function F is such that if "$\mathbf{x}$ *is preferred to* $\mathbf{y}$" with utilities $u_1, \ldots, u_n$, then "$\mathbf{x}$ *is also preferred to* $\mathbf{y}$" with utilities $\phi_1 \circ u_1, \ldots, \phi_n \circ u_n$?

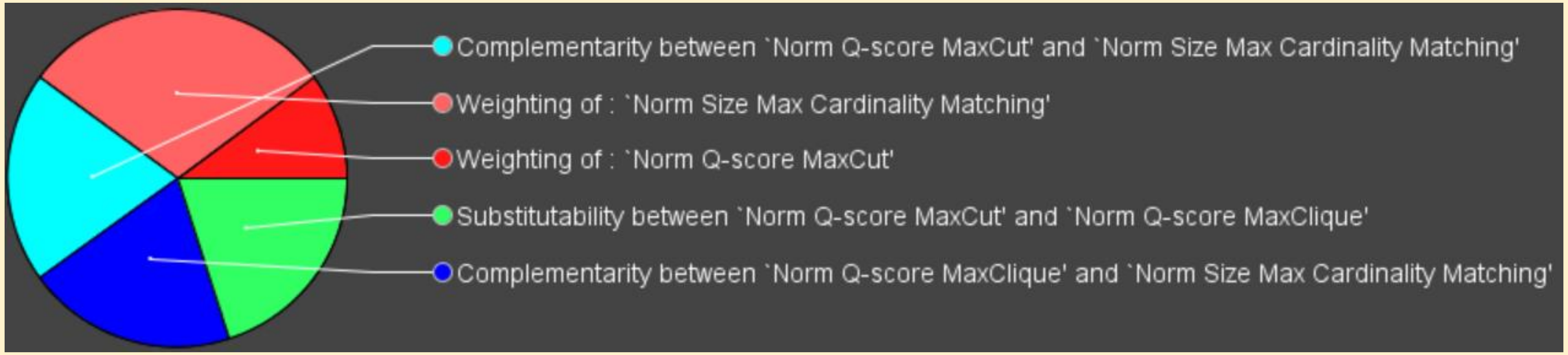
Elicitation with MYRIAD-Q



Elicitation with MYRIAD-Q



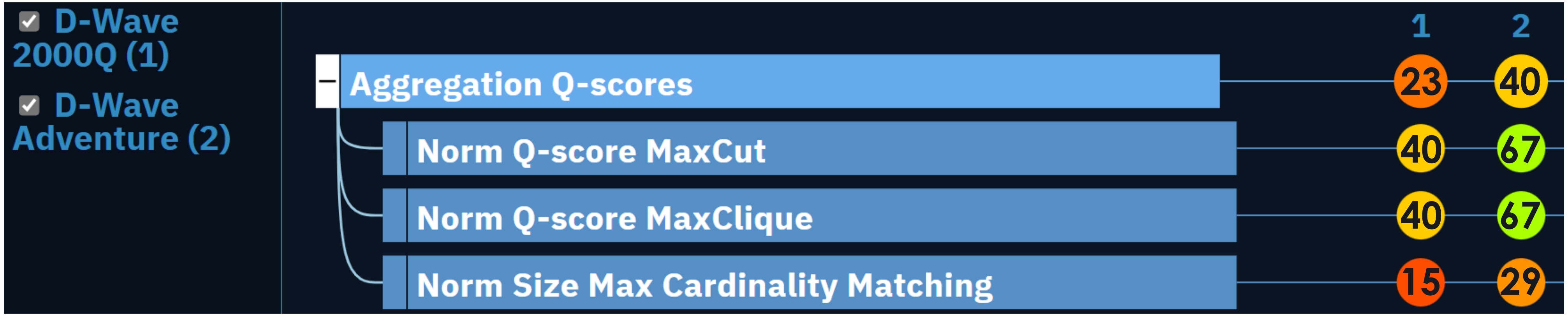




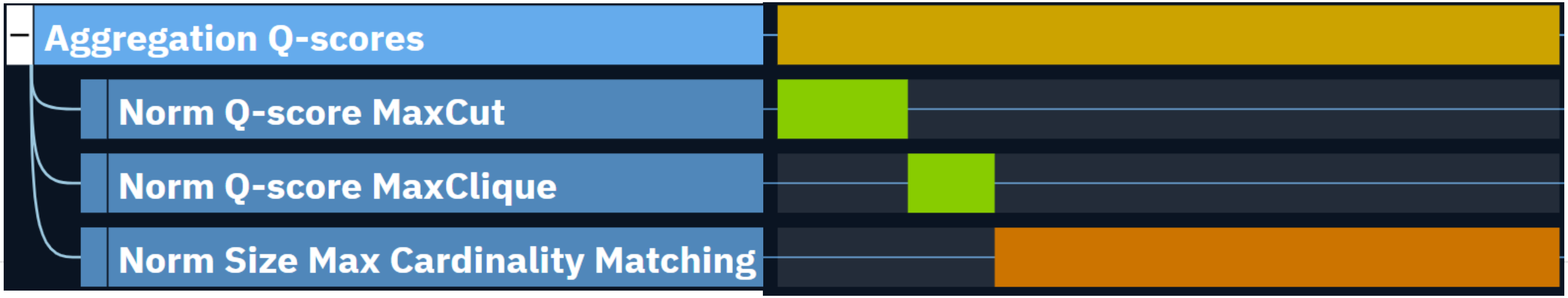
$$U = 0.3\, u_{MCM} + 0.1\, u_{MCut} + 0.2 \min(u_{MCM}, u_{MCut})$$
$$+ 0.2 \min(u_{MCM}, u_{MCli}) + 0.2 \max(u_{MCut}, u_{MCli})$$

Output of MCDA

	Q-score MaxCut	Q-score MaxClique	Max size MaxCardMatching
D-Wave 2000Q	70	70	61
D-Wave Advantage	140	110	116



- ❑ Explanation of D-Wave Advantage in comparison to the best possible alternative



Take-away

> Multi-Criteria Decision Analysis

- Many models & elicitation techniques
- Focus on weighted utility methods

> Notion of reference element

- # reference elements depends on the invariance property

# reference elements	Invariance condition	Elicitation
0	Invariance condition too strong: **NO MODEL**	
1	Strong invariance condition: **only the geometric mean**	
2	Standard Invariance condition: **many compensatory models**	Illustration of the elicitation process