Quantum MHT

(Multiple Hypotheses Tracking)

Paul Rousset-Rouard[1], Bing-Hong Teh[2], Jean-Marc Divanon[2]

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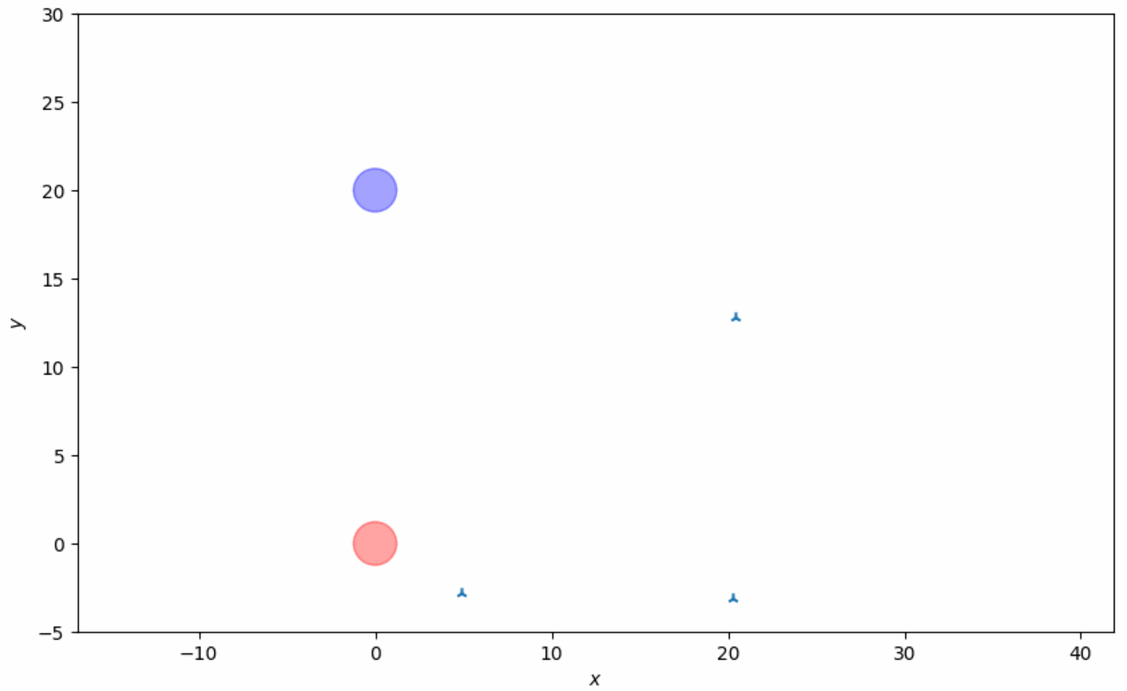

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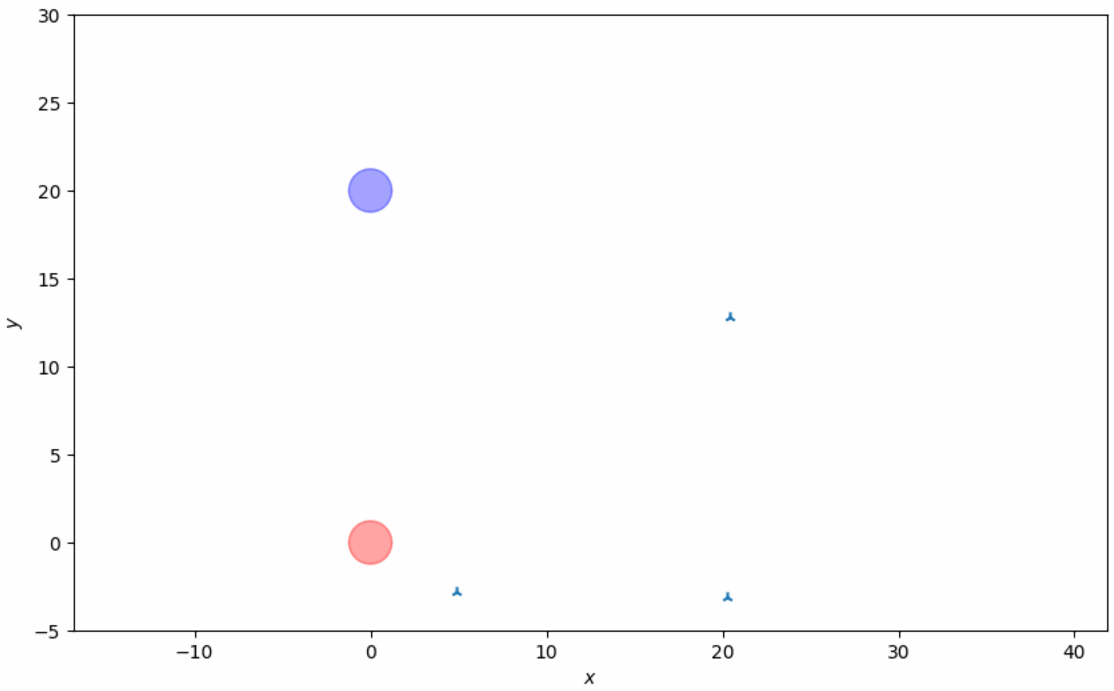
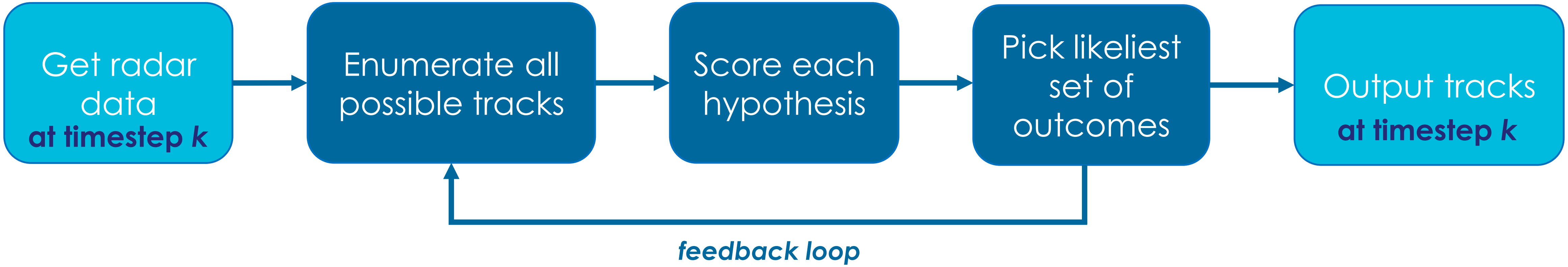
THE MHT ALGORITHM



Multiple Hypotheses Tracking (MHT)

> A well-known data-association method[1]

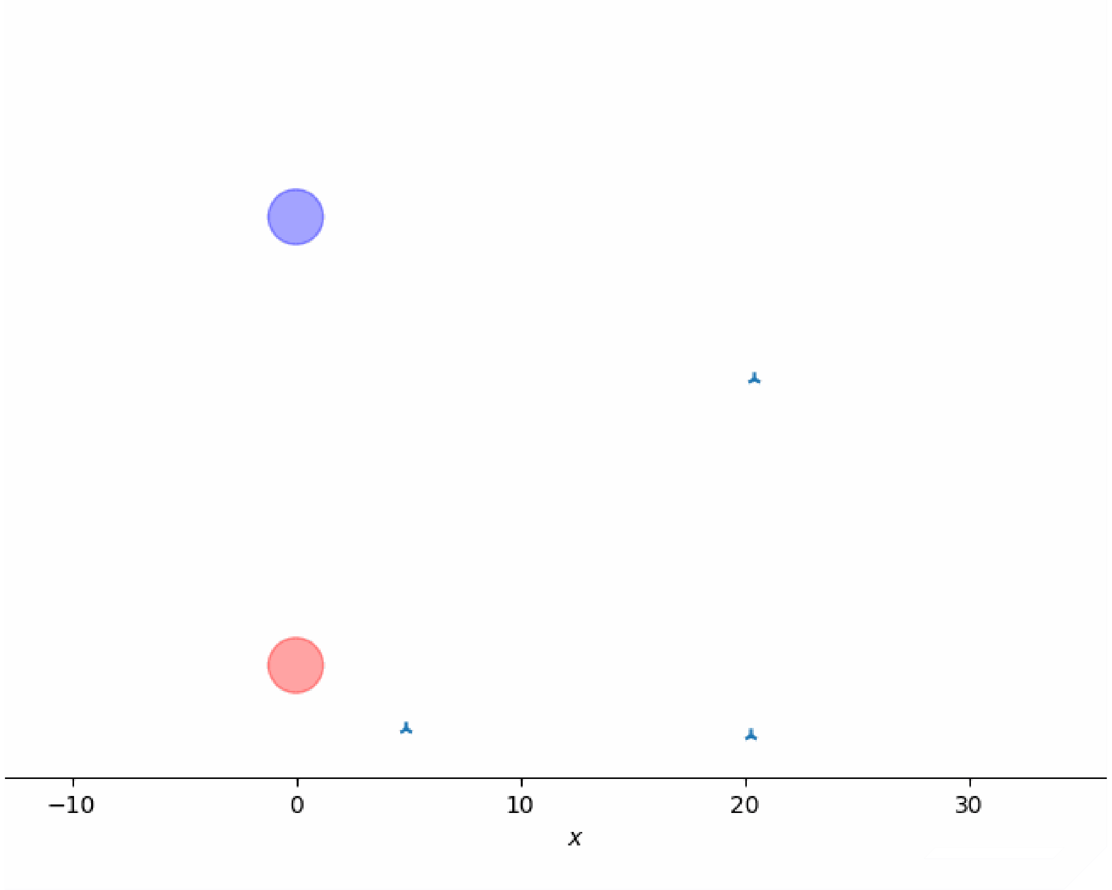


> Pruning techniques to avoid combinatorial explosion

+Kalman Filter
+Maximum likelihood scoring
+ …

[1] D. Reid. An algorithm for tracking multiple targets. IEEE, 1979.

MHT implementation



Example crossing

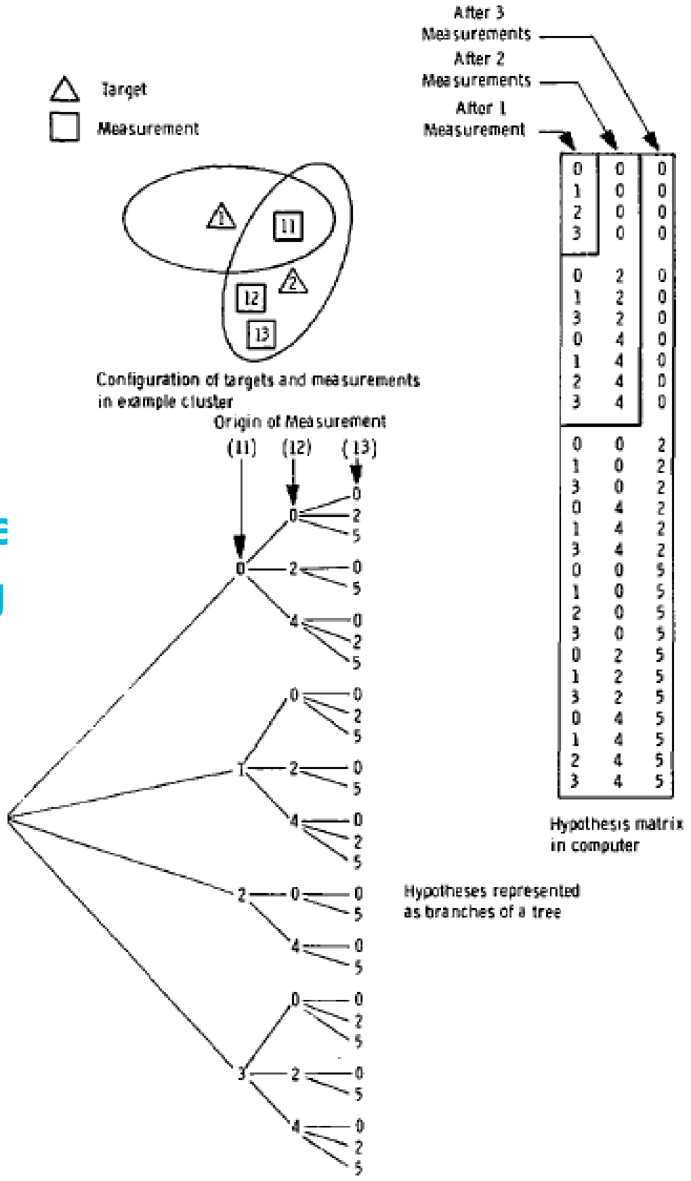
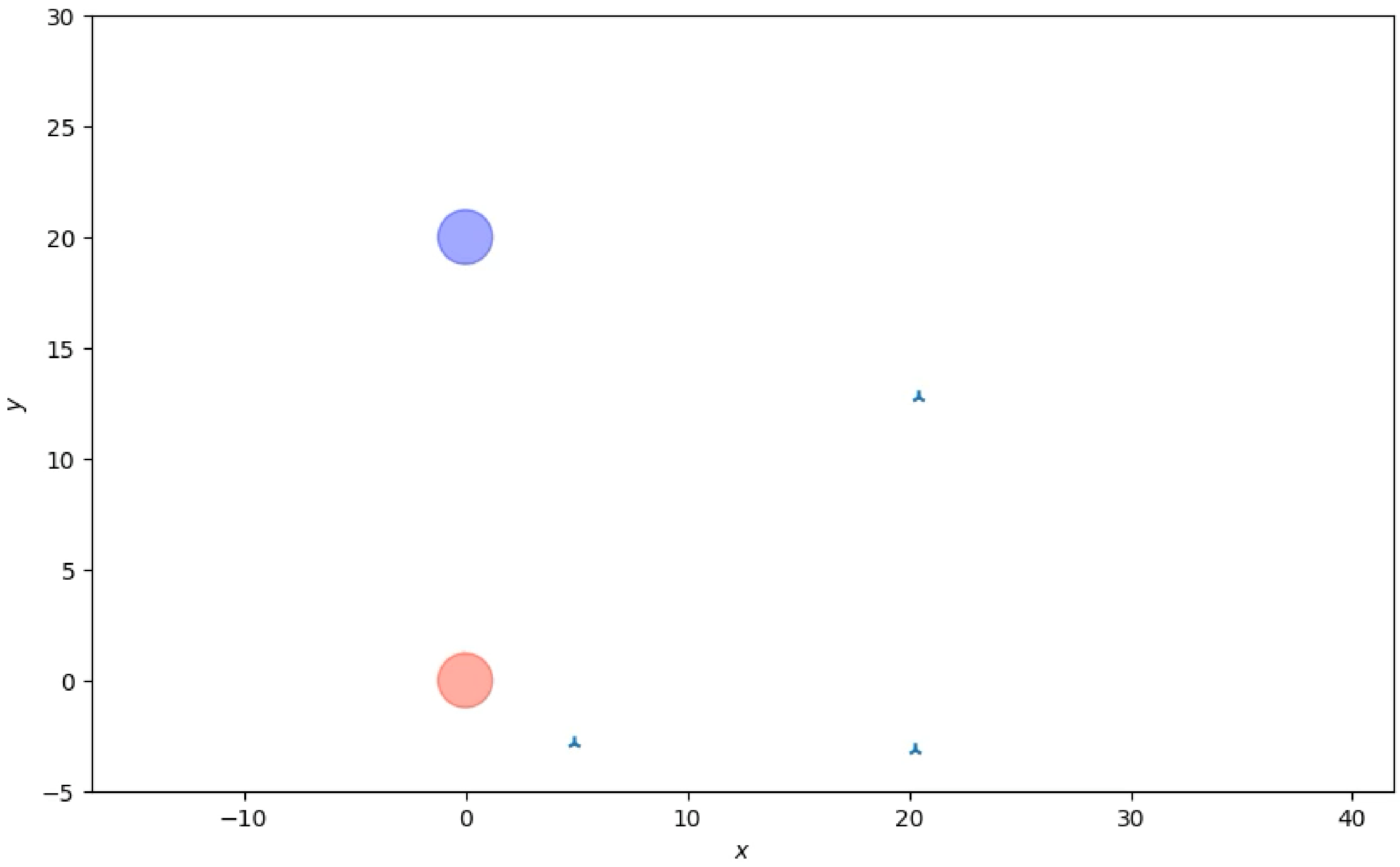
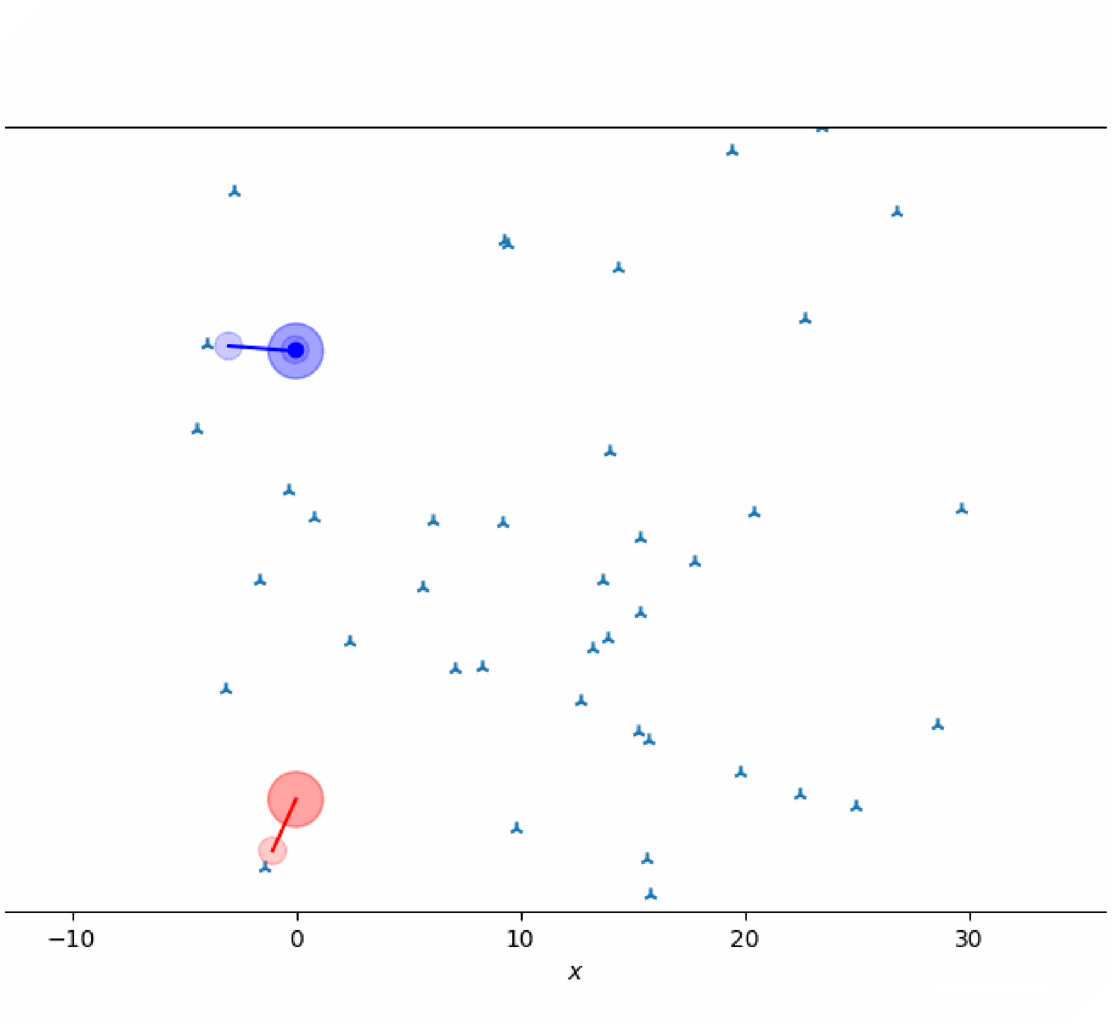


Figure 1: The hypotheses matrix (extracted from [1])

[1] D. Reid. An algorithm for tracking multiple targets. IEEE, 1979.

MHT implementation (slowed)





MHT implementation

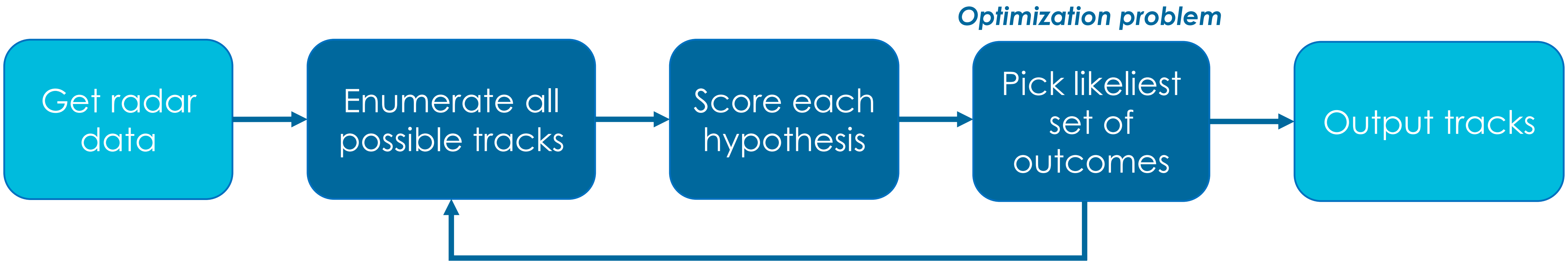
> Add more false alarms

> Tracks get lost

> Classical optimization algorithms do not yield good enough solution quality

Multiple Hypotheses Tracking (MHT)

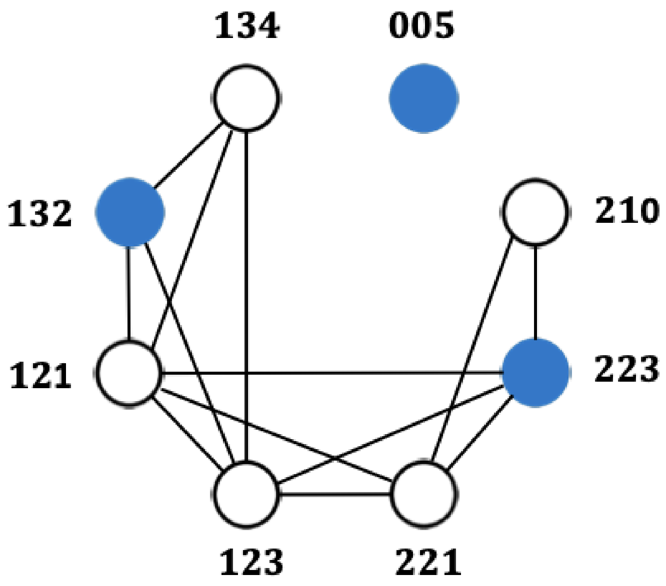
What about quantum solvers?



MWIS (Maximum Weighted Independent Set)

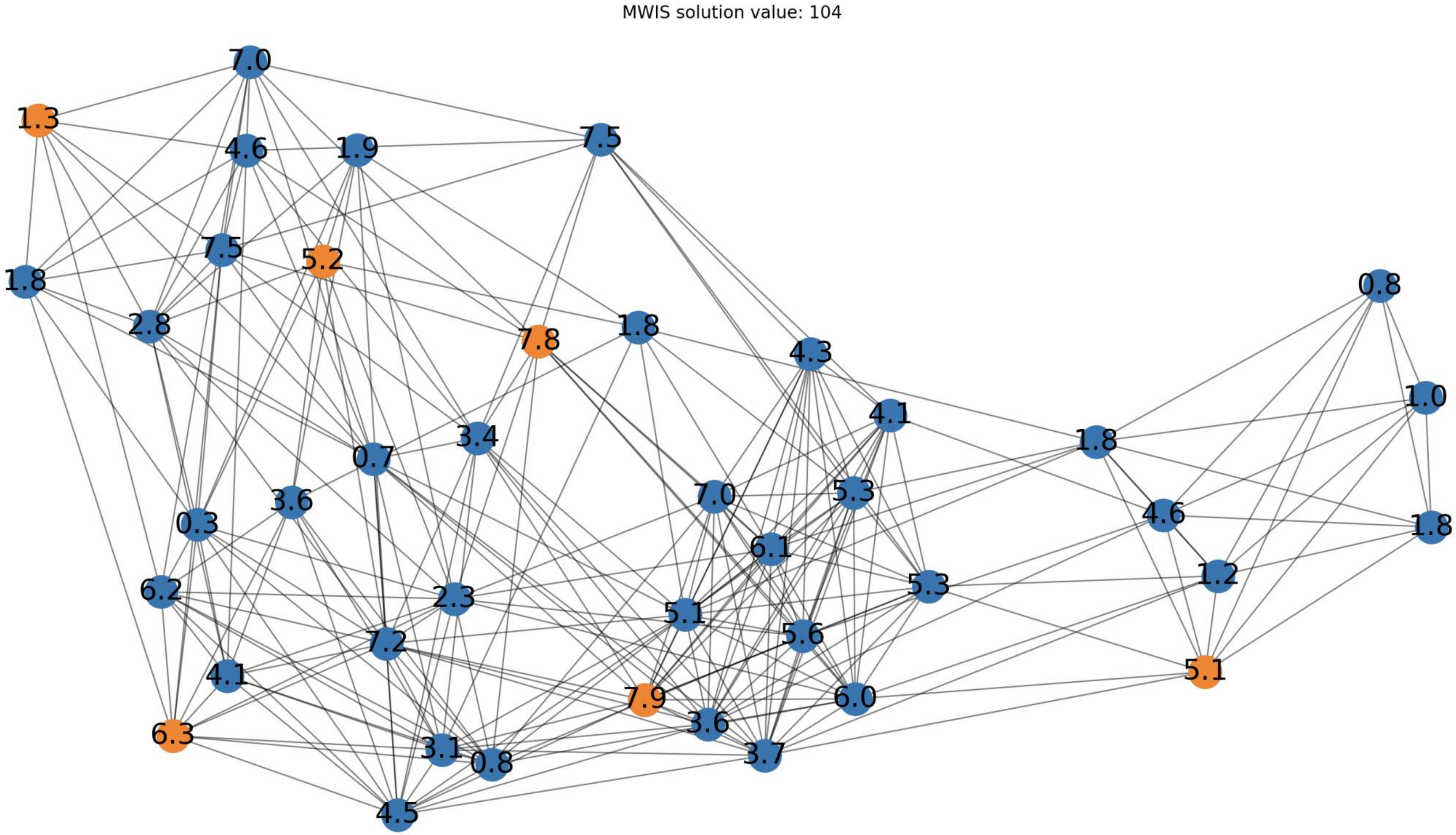
$$\max_{\mathbf{x}} \sum_{l} w_l x_l$$

$$\text{s.t.} \;\; x_l + x_j \leq 1, \;\; \forall (l, j) \in E, \;\; x_l \in \{0, 1\}.$$



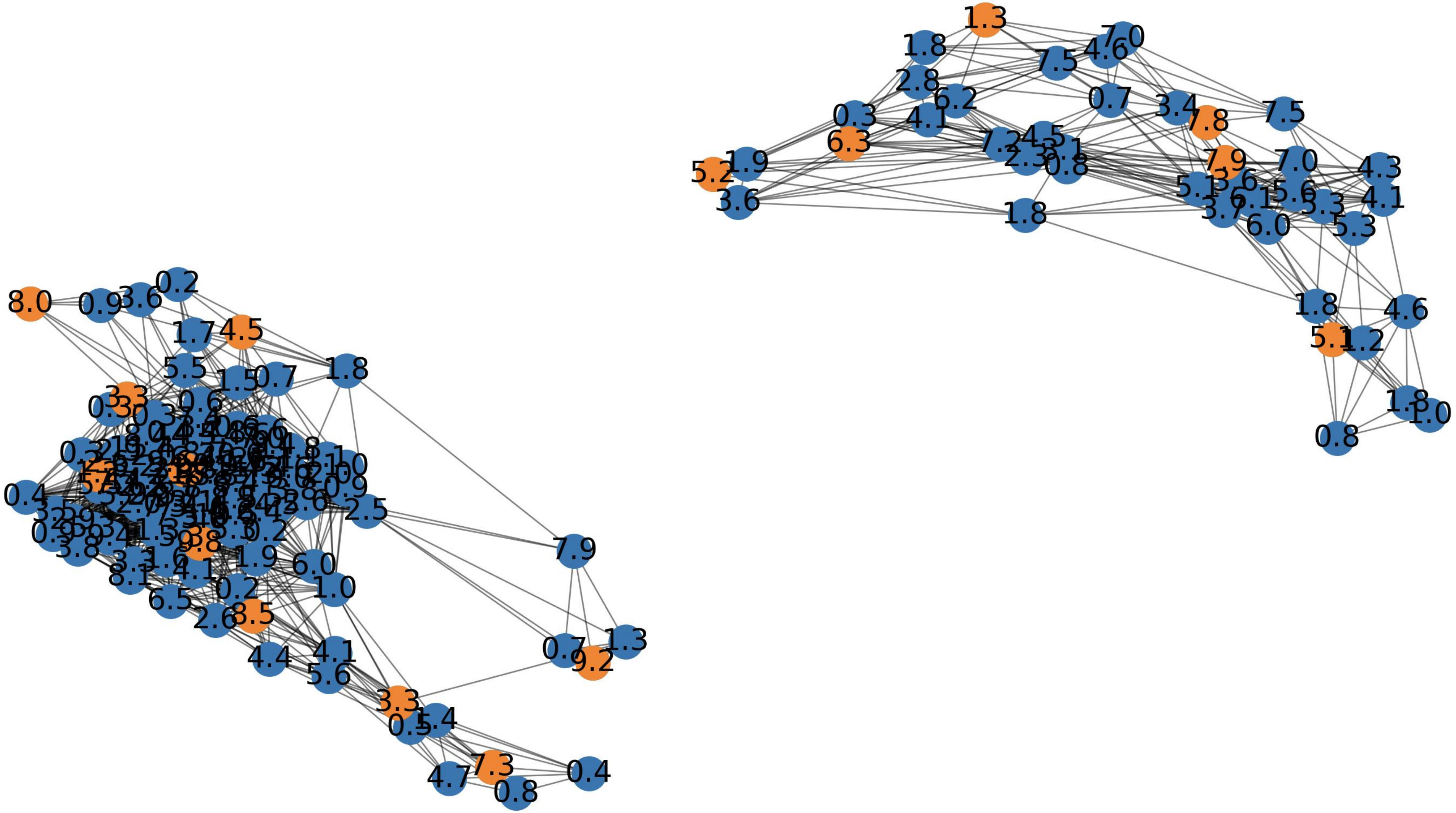
A. Ciptadi J. M. Rehg C. Kim, F. Li. Multiple hypothesis tracking revisited. IEEE, 2015.

An example of an MHT-derived graph



An example of an MHT-derived graph

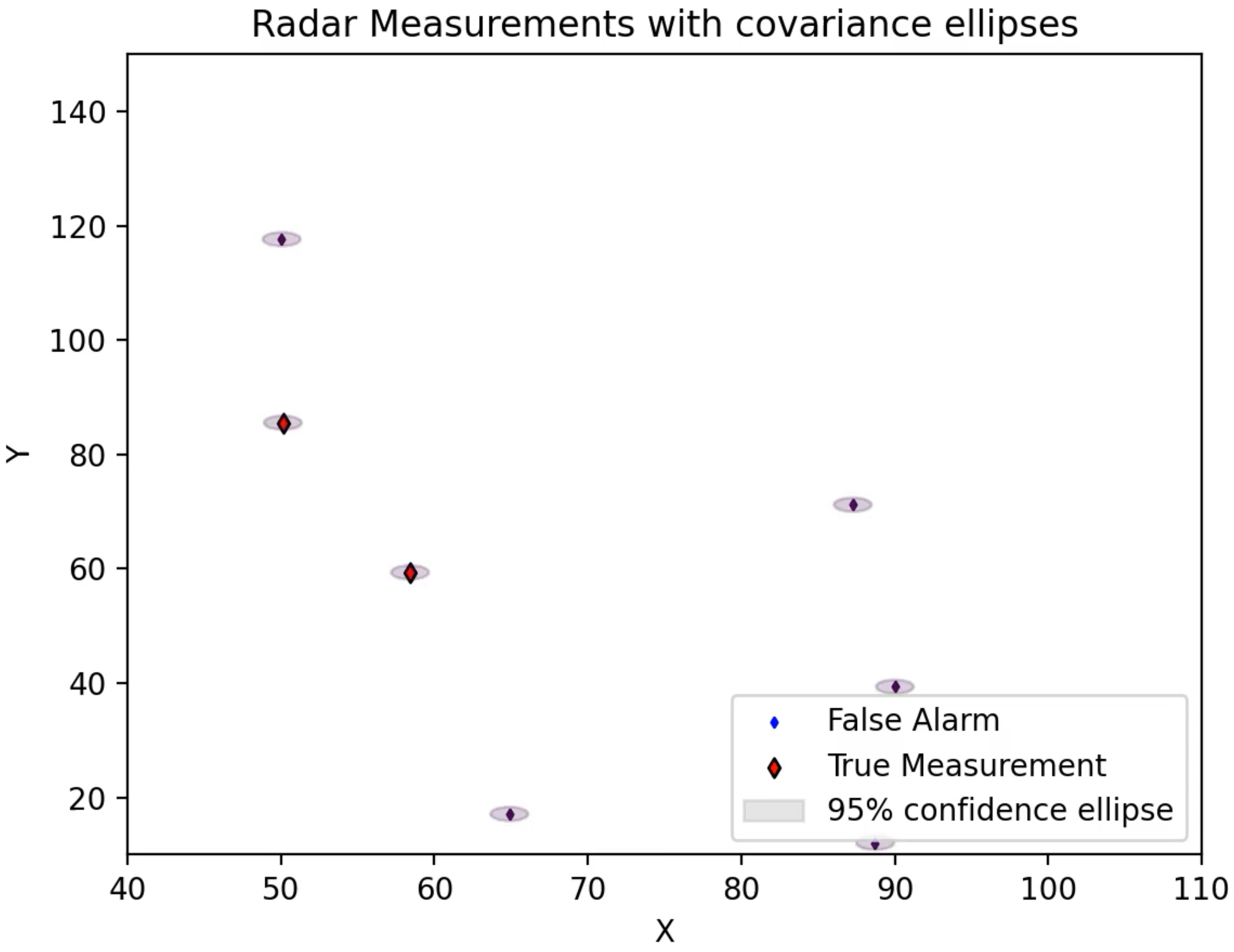
MWIS solution value: 104



Easy scenarios for MHT

> *Scenario "Three tracks"*

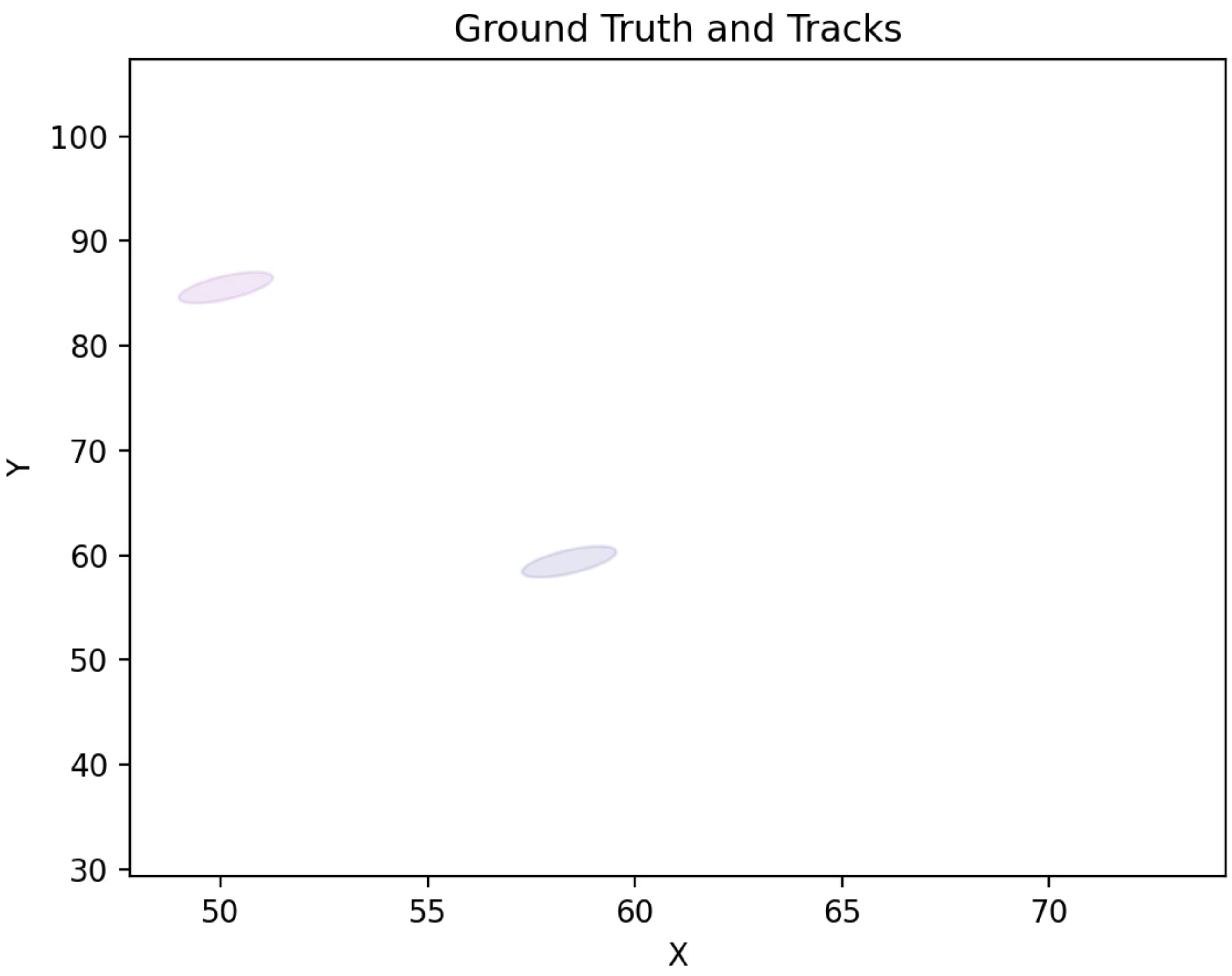
- ***Measurements***



Easy scenarios for MHT

> *Scenario "Three tracks"*

- *Outputs of the MHT algorithm*



Solvers implemented for MHT

Name	**TSM-WC**	**QUALEX-MS**	**SCIP**	**NetworkX**	**DWAVE**
Type	Classical	Classical	Classical	Classical	Hybrid
Exact ?	Yes	No	No	Yes	No
Strategy	Recursive backtracking	Motzkin-Straus QP	LP-relaxation	Branch & Bound	Lagrangian relaxation

5. Hicks Illya V. Warren, Jeffrey S. Combinatorial branch-and-bound for the maximum weight independent set problem, 2016.

6. Y. Liu F. Manya H. Jiang, C.-M. Li. A two-stage maxsat reasoning approach for the maximum weight clique problem, 2018.

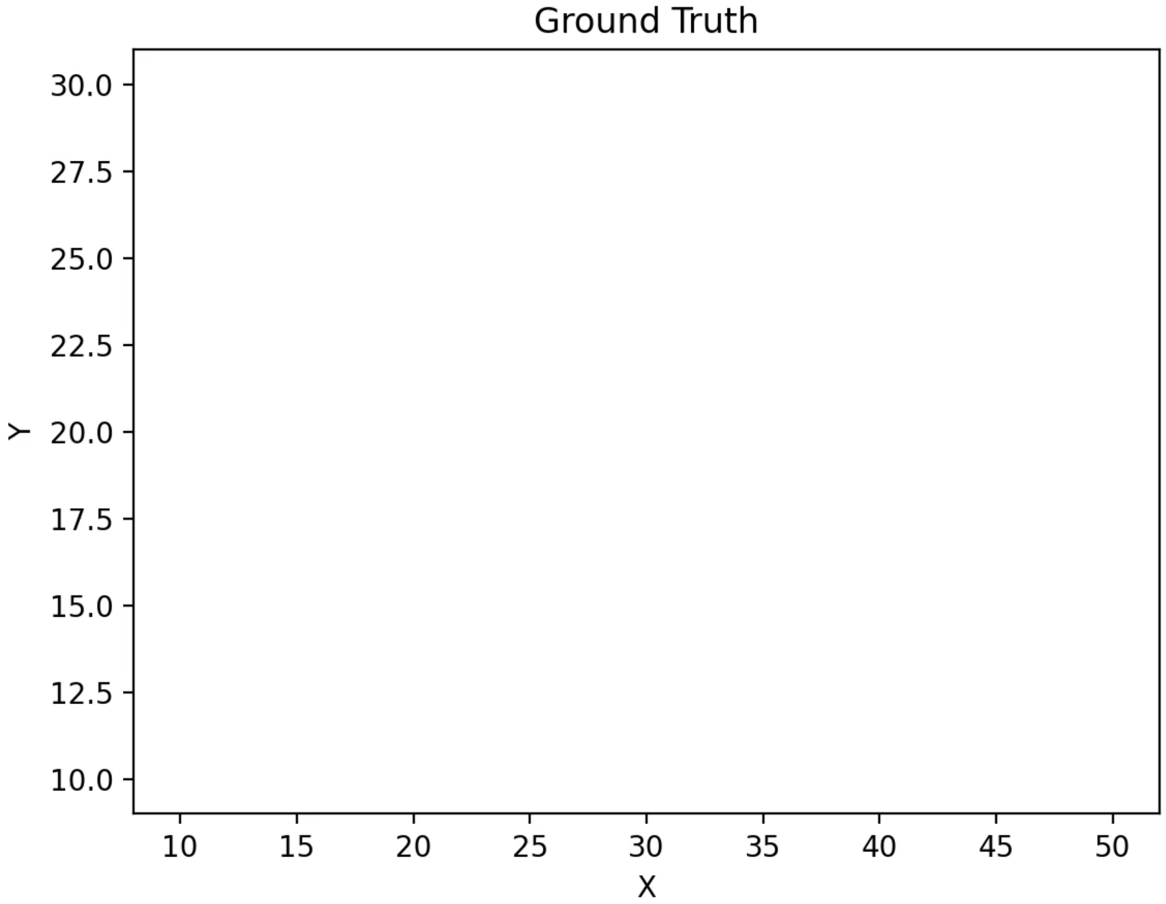
11. Stanislav Busygin, A new trust region technique for the maximum weight clique problem, 2006.

12. Sampo Niskanen and Patric R. J. Östergård, "Cliquer User's Guide, Version 1.0", 2003.

14. D-Wave, "Hybrid solvers for quadratic optimization," 2022.

Scenario for benchmarking

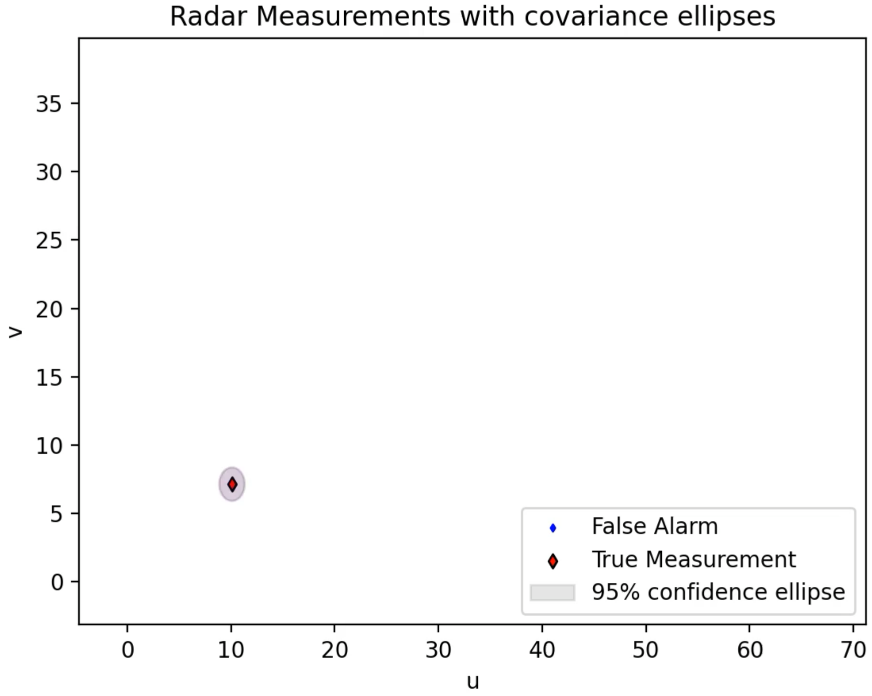
> *1 track going on a straight line, with a burst in false alarms for 8 frames*

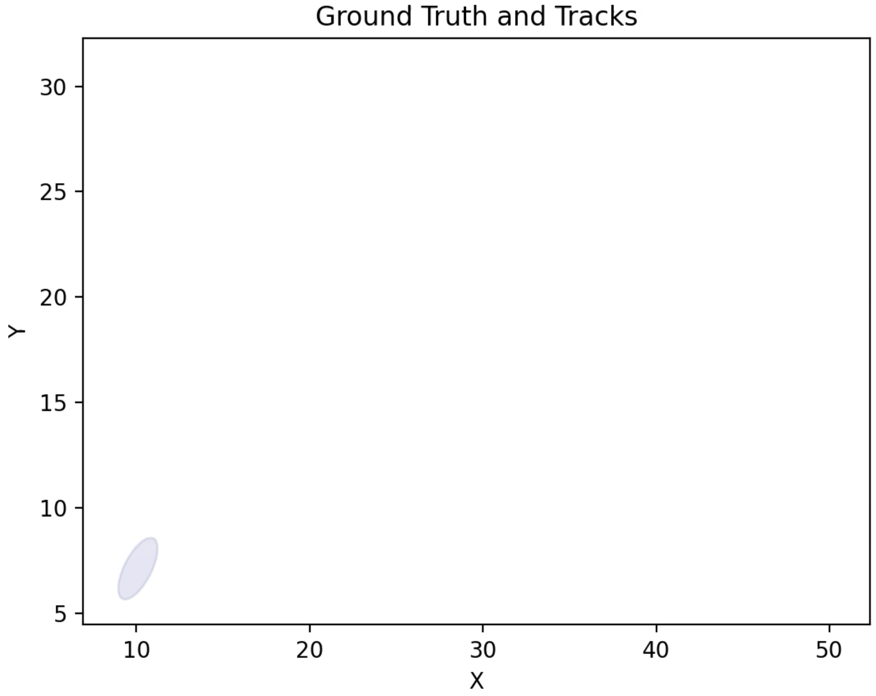


- 20 frames, $p_d = 0.9, \lambda_{clutter}$ $decupled\ from\ frame\ 3\ to\ 10$

Scenario for benchmarking

> *One easy example*



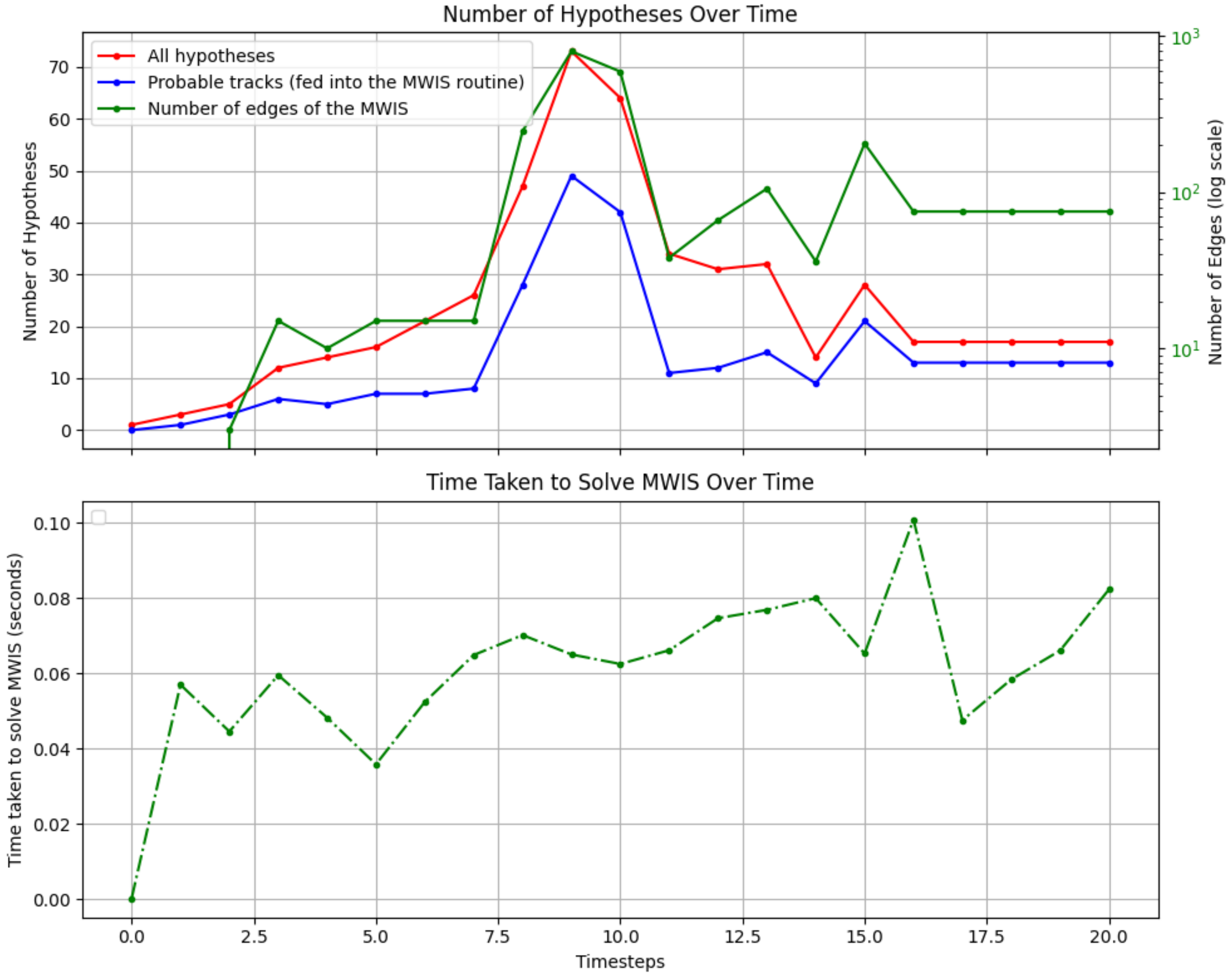


- *20 frames,* $p_d = 0.9, \lambda_{clutter}$ $decupled\ from\ frame\ 3\ to\ 10$

Scenario for benchmarking

> *One easy example, solver : TSM-WC*

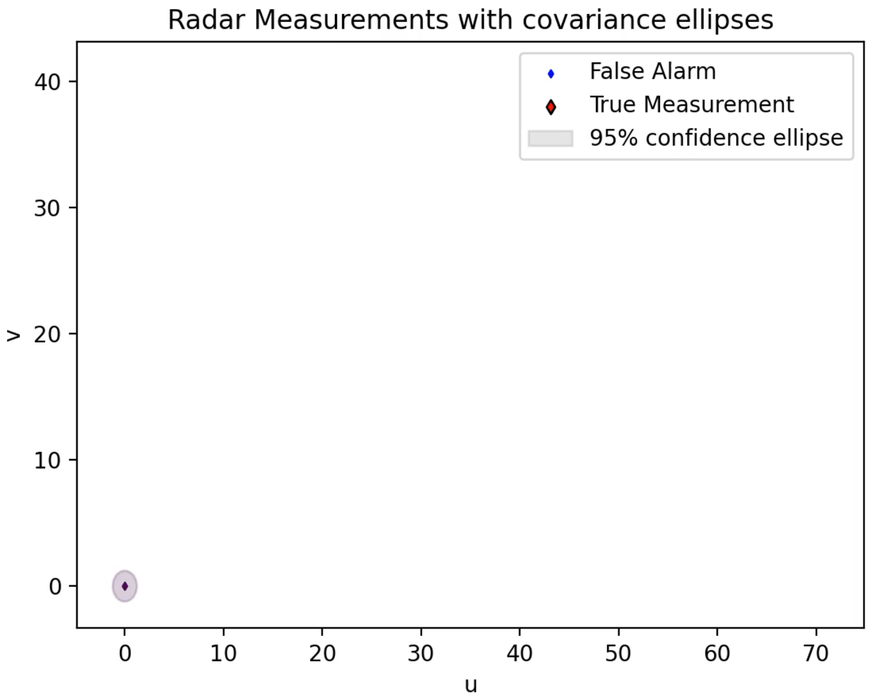
- $\lambda_{clutter} = 10^{-5}$

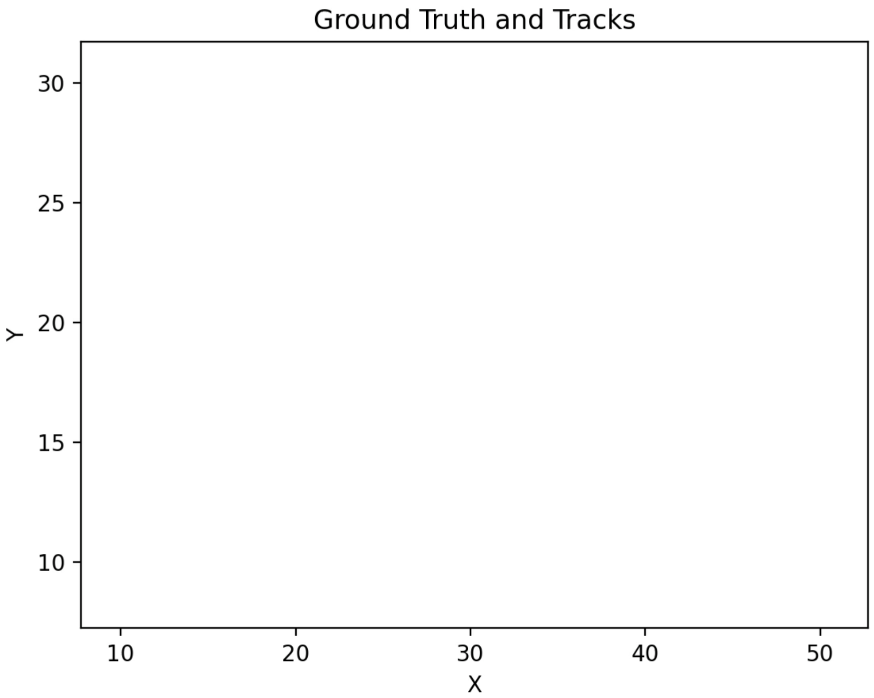


Scenario for benchmarking

> One hard example

- $\lambda_{clutter} = 10^{-4}$

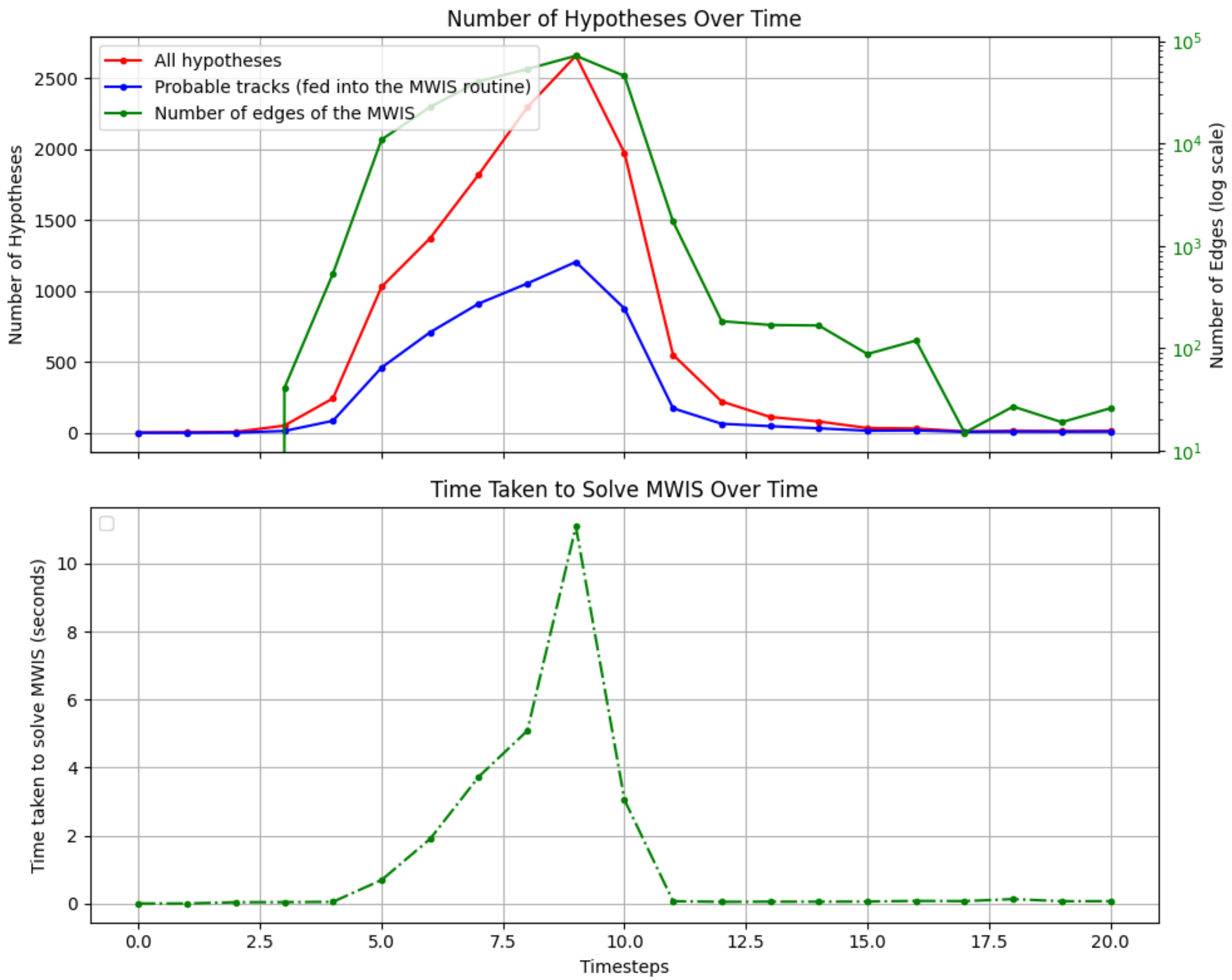




Scenario for benchmarking

> *One hard example, solver : TSM-WC*

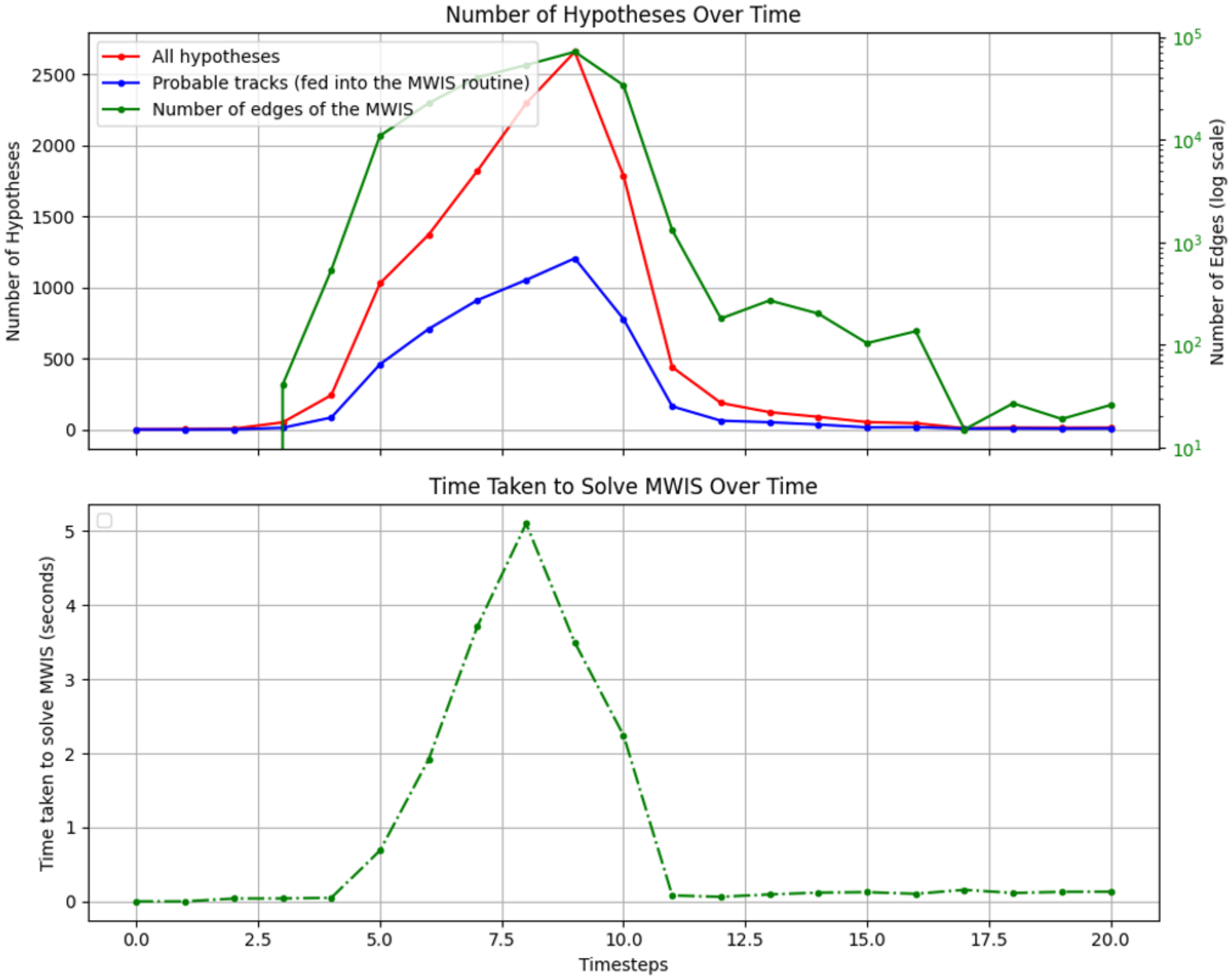
- $\lambda_{clutter} = 10^{-4}$



Scenario for benchmarking

> *Solver : TSM-WC and DWAVE*

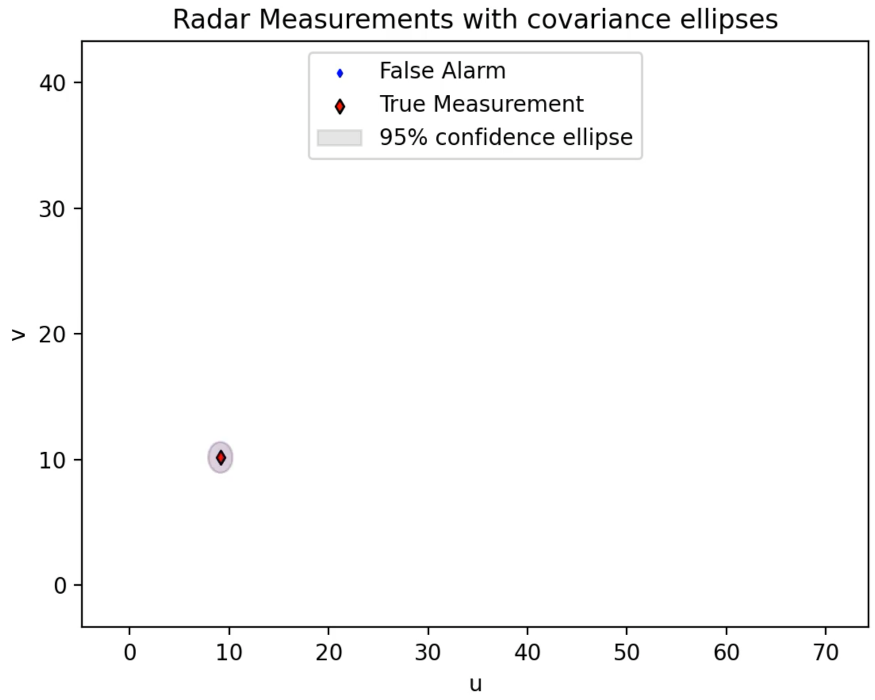
- $\lambda_{clutter} = 10^{-4}$
- *DWAVE solver is called at frame 9*

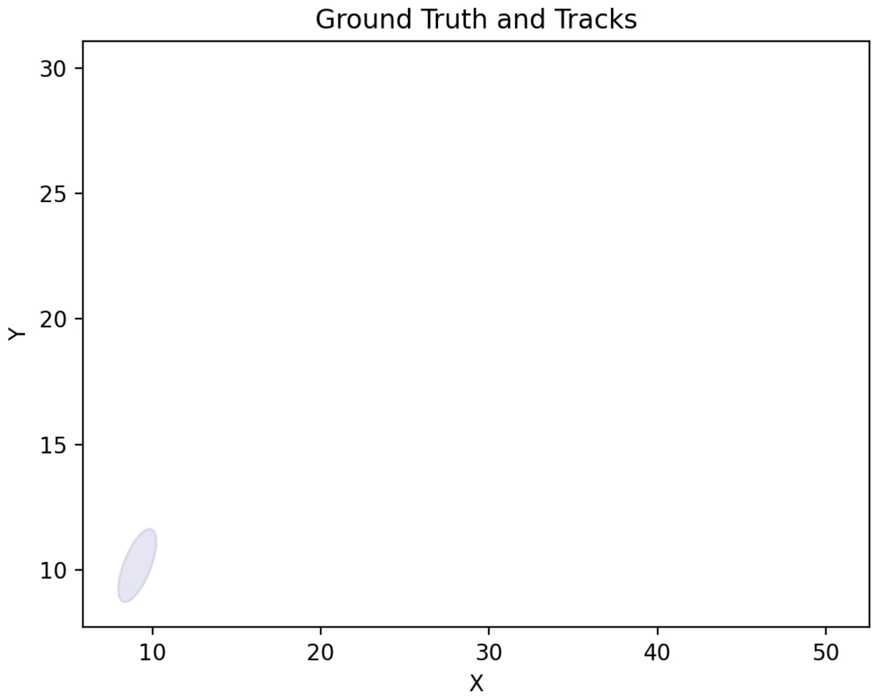


Scenario for benchmarking

> One harder example

- $\lambda_{clutter} = 1.1 \cdot 10^{-4}$

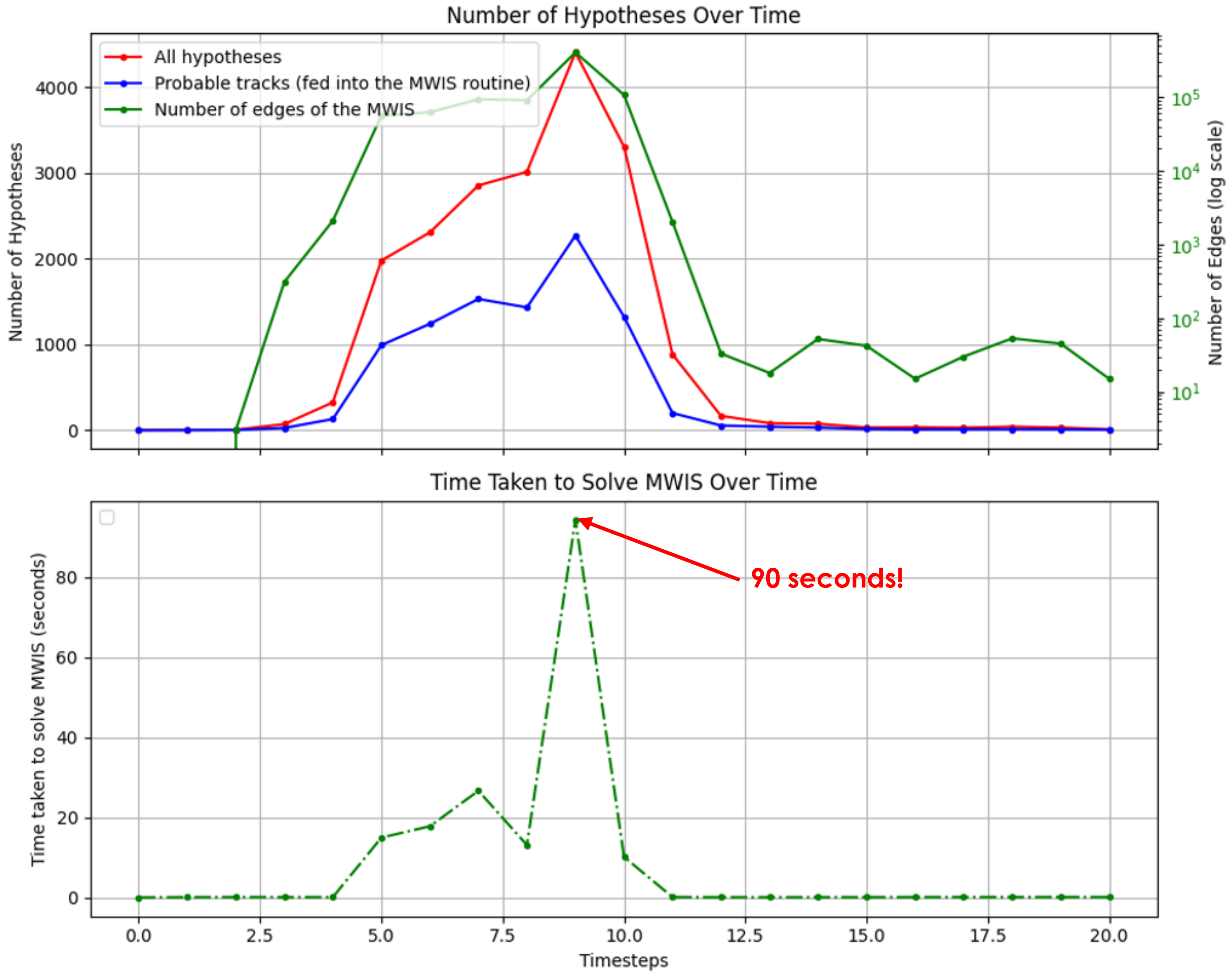




Scenario for benchmarking

> *One harder example, solver : TSM-WC*

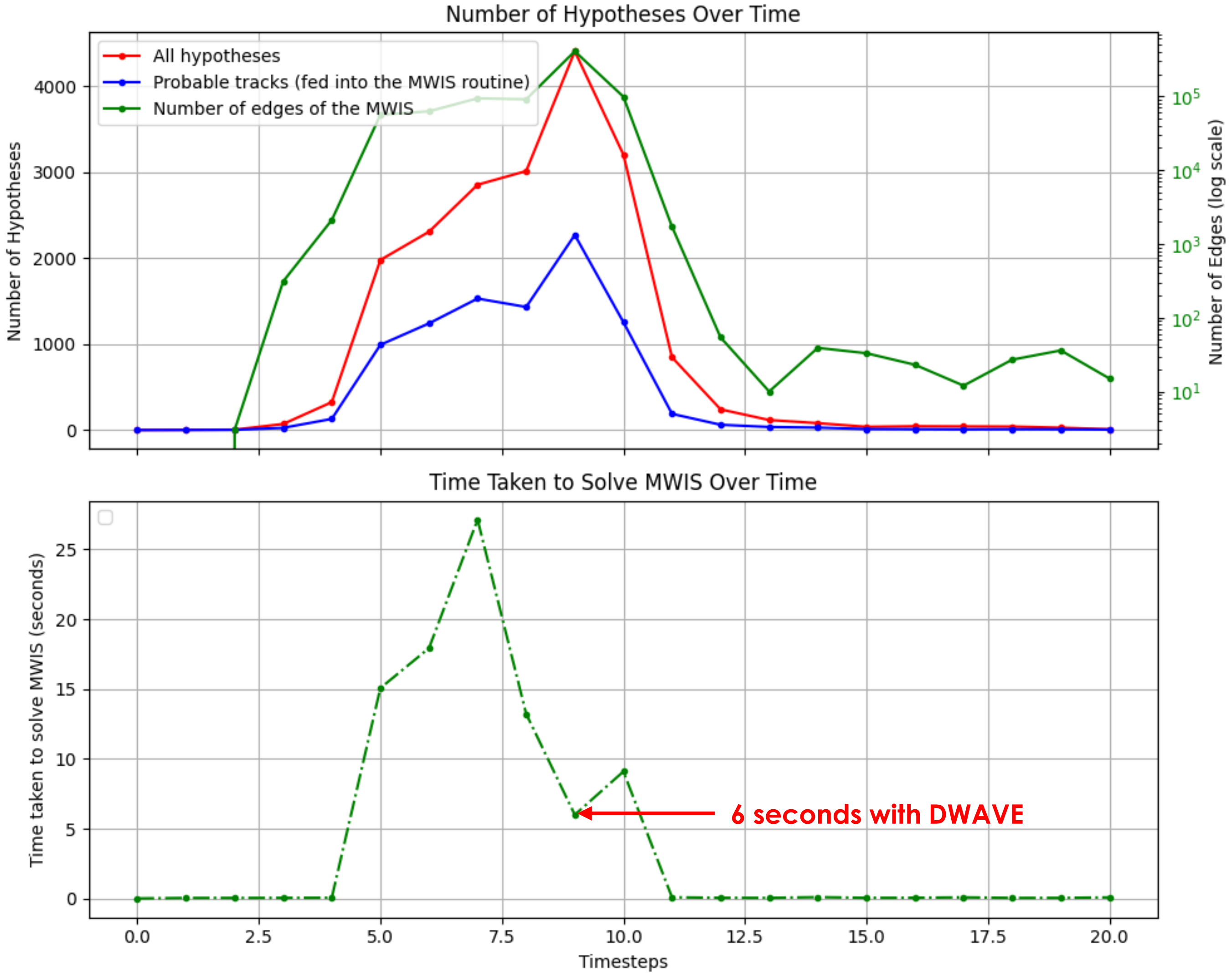
- $\lambda_{clutter} = 1.1 \cdot 10^{-4}$



Scenario for benchmarking

> *Solver : TSM-WC and DWAVE*

- $\lambda_{clutter} = 1.1 \cdot 10^{-4}$
- *DWAVE solver is called at frame 9*



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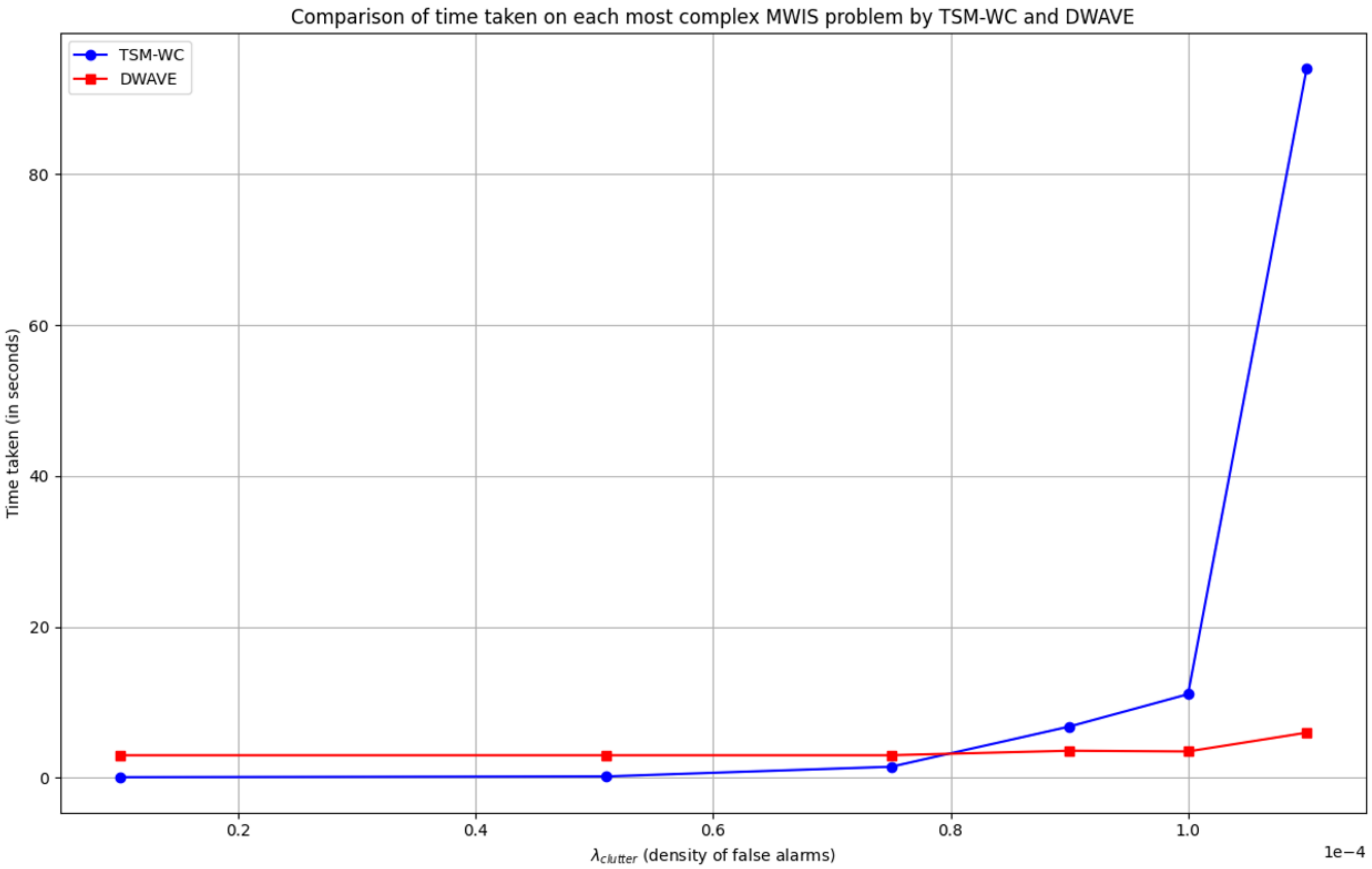
DWAVE vs TSM-WC

DWAVE's hybrid solver has a minimal runtime of 3 seconds



Conclusion

> ***Quantum annealing becomes interesting when*** $\lambda_{clutter} > 9.10^{-5}$

- *As false alarms increase, quantum annealing becomes clearly the best option. Up to* ***15 times slower*** *!*

> ***However, it still needs to be coupled with a classical solver to yield overall good solution quality (and find the correct track)***

> C12 real HW execution time

				Theoretical execution time on real C12 HW		
$\lambda_{clutter}$	#hypothesis	TSM-WC [s]	D-Wave [s]	Callisto annealing [us]	Callisto (best) [ms]	Callisto (worst) [ms]
10^-5	74	0.1	3	10	5 - 10	10.084
5 x 10^-5	528	0.18	3	10		10.538
7.5 x 10^-5	1900	2.83	3	150		12.1
9 x 10^-5	2019	6.2	3.6	150		12.17
10^-4	3979	39.21	3.5	150		14.129
1.1 x 10^-4	4180	73.63	6	150		14.55

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Thank you

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Any questions ?

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not disclosed

not disclosed

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